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COERCIVITY CONDITIONS AND ZEROS OF ACCRETIVE OPERATORS

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Abstract. We present some coercivity conditions for accretive operators, which are the modified versions of the coercivity conditions in the literature for monotone operators in Hilbert and Banach spaces. We prove that the non-emptiness and boundedness of the zero set of an m-accretive operator are equivalent to these coercivity conditions.
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1. Introduction

1.1. **Preliminaries.** Let X be a Banach space with norm $\|\cdot\|$ and dual X^*. We denote the value of $x^* \in X^*$ at $x \in X$ by $\langle x, x^* \rangle$. A Banach space X is called separable if it contains a countable, dense subset. In separable Banach spaces, the closed unit ball of X^* is metrizable in weak* topology, therefore by Banach-Alaoglu theorem it is sequentially compact. Let $X^{**} = (X^*)^*$ denote the second dual of X. The canonical map $i : X \underset{x \mapsto \hat{x}}{\rightarrow} X^{**}$, which $\hat{x}(f) = f(x),\ f \in X^*$ gives a linear isomorphism (embedding) from X into X^{**}. X is called reflexive if $i : X \to X^{**}$ is surjective. A Banach space X is said to be strictly convex if $\|\frac{x+y}{2}\| < 1$, for all $x, y \in X$ with $\|x\| = \|y\| = 1$ and $x \neq y$. Equivalently X is strictly convex if and only if for every $x, y \in X$ that are linearly independent, we have $\|x+y\| < \|x\| + \|y\|$. $J : X \longrightarrow 2^{X^*}$ defined by

$$J(x) := \{x^* \in X^* : \langle x, x^* \rangle = \|x\|^2, \|x\| = \|x^*\|\}$$

is called the duality mapping from X into X^*. Reflexivity of X implies that the duality mapping $J : X \to 2^{X^*}$ is surjective. See [3, 4, 20].

A multi-valued operator $A : X \longrightarrow 2^X$ with domain $D(A) := \{z \in X : Az \neq \emptyset\}$ and range $R(A) := \bigcup\{Az : z \in D(A)\}$ is said to be accretive if for each $x_i \in D(A)$ and $y_i \in Ax_i, i = 1, 2$, there is $j \in J(x_1 - x_2)$ such that $\langle y_1 - y_2, j \rangle \geqslant 0$. In this paper in order to simplify we write $\langle y_1 - y_2, J(x_1 - x_2) \rangle \geq 0$. An accretive operator

A is said to be maximal if its graph $G(A) = \{(x, y) \in X \times X : x \in D(A),\ y \in Ax\}$ is not properly contained in the graph of any other accretive operator. An accretive operator A is called m-accretive if for some $\lambda > 0$ (equivalently, for each $\lambda > 0$) $R(I + \lambda A) = X$, where I is the identity operator from X to X. It is well-known that, every m-accretive operator is maximal accretive. For an m-accretive operator A and for all $x_0 \in X$ and $\lambda > 0$, the inclusion $0 \in (x - x_0) + \lambda Ax$ has a unique solution. This unique point is denoted by $\mathcal{J}_\lambda x_0 := (I + \lambda A)^{-1}(x_0)$ and is called the resolvent of A of order λ at the point x_0. It is easily seen that $\mathrm{Fix}(\mathcal{J}_\lambda) = A^{-1}(0)$, where $\mathrm{Fix}(\mathcal{J}_\lambda) = \{x \in X : \mathcal{J}_\lambda(x) = x\}$. It is well-known that for an m-accretive operator A, its resolvent of order λ is nonexpansive for each $\lambda > 0$; i.e.

$$\|\mathcal{J}_\lambda x - \mathcal{J}_\lambda y\| \leq \|x - y\|, \quad \forall x, y \in X. \tag{1.1}$$

For each m-accretive operator A and $\lambda, \mu > 0$, the resolvent identity

$$\mathcal{J}_\lambda x = \mathcal{J}_\mu\Big(\frac{\mu}{\lambda}x + \frac{\lambda - \mu}{\lambda}\mathcal{J}_\lambda x\Big), \quad \forall x \in X \tag{1.2}$$

holds. For accretive operators, their properties and resolvents the reader can consult with [4, 20].

We consider the inclusion problem, which is to find $x \in X$ such that

$$0 \in A(x), \tag{1.3}$$

where $A : X \longrightarrow 2^X$ is an m-accretive operator. The solution set of the inclusion problem (1.3) is denoted by $A^{-1}(0) = \{x \in D(A) : 0 \in A(x)\}$. In the last subsections of this section, we give some motivations for studying (1.3) as well as some coercivity conditions are stated which are needed for studying (1.3). Section 2 is devoted to the main results of the paper. We prove the equivalency of the coercivity conditions (A$''$) and (B$''$)(see Subsection 1.3) with non-emptiness and boundedness of the zero set of the m-accretive operator A. In Section 3, we apply Theorem 2 to prove the existence of a fixed point for nonexpansive mappings in Banach spaces.

1.2. **Some Motivations.** Finding a solution of (1.3) for an accretive operator A is applicable in fixed point theory and evolution equations of accretive type.

Fixed Point Theory. Let X be a Banach space and $C \subset X$ be nonempty and closed. Let $T : C \to X$ satisfy

$$\langle Tx - Ty, j\rangle \leq \|x - y\|^2, \tag{1.4}$$

for each $x, y \in C$ and some $j \in J(x-y)$, then $A := I - T$ is an accretive operator and $\mathrm{Fix}(T) = A^{-1}(0)$. Evidently, every nonexpansive mapping (i.e. $\|Tx - Ty\| \leq \|x - y\|$) satisfies (1.4). An easy application of Banach contraction principle implies that if $T : X \to X$ is nonexpansive, then $A = I - T$ is m-accretive. For more details see Theorem 4.6.4 in [20].

Evolution Equations of Accretive Type. Consider the first order evolution equation

$$\begin{cases} -u'(t) \in A(u(t)), & \text{a.e. } t \geq 0 \\ u(0) = x \in \overline{D(A)} \end{cases} \tag{1.5}$$

where $A : X \to 2^X$ is an m-accretive operator. If $A^{-1}(0) \neq \emptyset$, say $x \in A^{-1}(0)$, the differential inclusion (1.5) has a stationary solution $u(t) \equiv x$. By Theorem 2.1 in [1, Chap. 3 p. 124] every solution to (1.5) is bounded and stable. Existence of a solution to the stationary equation (1.3) also ensures the existence of solutions to the following incomplete Cauchy problem associated with A and its discrete version:

$$\begin{cases} u''(t) \in Au(t), & \text{a.e. } t \geq 0 \\ u(0) = x \in \overline{D(A)}, & \sup_{t\geq 0} \|u(t)\| < +\infty. \end{cases} \tag{1.6}$$

For a fuller treatment we refer the reader to [16, 17].

1.3. **Coercivity Conditions.** The problem of the existence of a solution for (1.3), when A is an accretive operator, is taken into consideration by many authors, see for instance [6, 7, 8, 9, 11, 12, 13, 14, 15, 18, 19]. In this paper, we consider the problem (1.3) by assuming the coercivity like conditions studied in [2, 5, 10, 21] for maximal monotone operators. In the next definition, we present some coercivity conditions that were either mentioned in the literature for monotone operators (Conditions (A) and (B)) or to the best of our knowledge seem new (Conditions (A″) and (B″)).

Definition 1.1 Let $A : X \to 2^X$ be an accretive operator. Following are some coercivity conditions for A:

- (A) There exists $\rho > 0$ such that for every $x \in D(A)$ with $\|x\| > \rho$ there is $\bar{x} \in D(A)$ with $\|\bar{x}\| < \|x\|$ satisfying: $\inf_{y\in A(x)} \langle y, J(x - \bar{x})\rangle \geqslant 0$.
- (B) There exists $\rho > 0$ such that for every $x \in D(A)$ with $\|x\| > \rho$ there is $\bar{x} \in D(A)$ with $\|\bar{x}\| < \|x\|$ satisfying: $\sup_{\bar{y}\in A(\bar{x})} \langle \bar{y}, J(x - \bar{x})\rangle > 0$.
- (A′) There exist $\rho > 0$ and $z \in X$ such that for every $x \in D(A)$ with $\|x - z\| > \rho$ there is $\bar{x} \in D(A)$ with $\|\bar{x}-z\| < \|x-z\|$ satisfying: $\inf_{y\in A(x)} \langle y, J(x-\bar{x})\rangle \geqslant 0$.
- (B′) There exist $\rho > 0$ and $z \in X$ such that for every $x \in D(A)$ with $\|x - z\| > \rho$ there is $\bar{x} \in D(A)$ with $\|\bar{x}-z\| < \|x-z\|$ satisfying: $\sup_{\bar{y}\in A(\bar{x})} \langle \bar{y}, J(x-\bar{x})\rangle > 0$.
- (A″) There exist $\rho > 0$ and $z \in X$ such that for every $x \in D(A)$ with $\|x - z\| > \rho$ there is $\bar{x} \in D(A)$ with $\|\bar{x} - z\| < \frac{1}{2}\|x - z\|$ satisfying:
$$\inf_{y\in A(x)} \langle y, J(x - \bar{x})\rangle \geqslant 0.$$
- (B″) There exist $\rho > 0$ and $z \in X$ such that for every $x \in D(A)$ with $\|x - z\| > \rho$ there is $\bar{x} \in D(A)$ with $\|\bar{x} - z\| < \frac{1}{2}\|x - z\|$ satisfying:
$$\sup_{\bar{y}\in A(\bar{x})} \langle \bar{y}, J(x - \bar{x})\rangle > 0.$$

Conditions (A) and (B) of 1.3.1 are the modified versions of the coercivity conditions (A) and (B) defined in [21], which have been applied to study (1.3) for maximal monotone operators in Hilbert and Banach spaces [2, 5, 10, 21].

Proposition 1.2 *The following implications hold for the above coercivity conditions:*

$$\begin{array}{ccccc} A'' & \Longrightarrow & A & \Longrightarrow & A' \\ \Uparrow & & \Uparrow & & \Uparrow \\ B'' & \Longrightarrow & B & \Longrightarrow & B' \end{array}$$

Proof. $A'' \Rightarrow A$: Take $z \in X$ and $\rho > 0$ as assumed in A''. A simple computation shows that $\rho_0 = 3\|z\| + \rho$ satisfies the coercivity condition A. $A \Rightarrow A'$ is trivial. Similar proof works for implications $B'' \Rightarrow B \Rightarrow B'$. $B'' \Rightarrow A''$ is proved while proving Theorem 2. $B \Rightarrow A$ and $B' \Rightarrow A'$ are similar.

2. Main results

In this section we state the main results, which prove that the existence and boundedness of solutions to (1.3) are respectively equivalent to the coercivity conditions (A$''$) and (B$''$). These results extend the results of [21] to accretive operators in Banach spaces. In Theorem 2 we remove the extra condition "convexity of the domain of the operator" assumed in [21, Theorem 4.1]. Our approach is using the resolvent operator.

Theorem 2.1 *Let X be a reflexive and separable Banach space. Suppose $A : D(A) \subseteq X \longrightarrow 2^X$ is an m-accretive mapping. Then the coercivity condition* (A$''$) *in Definition 1.1 holds if and only if the inclusion problem* (1.3) *has a solution.*

Proof. **Only if part**. Suppose the coercivity condition (A$''$) holds. Let $\rho > 0$ and $z \in X$ as assumed in the coercivity condition (A$''$). Since A is m-accretive, then $x_\lambda := \mathcal{J}_\lambda(z) = (I + \lambda A)^{-1}(z) \in D(A)$ or $z \in x_\lambda + \lambda A x_\lambda$, hence $\frac{-1}{\lambda}(x_\lambda - z) \in Ax_\lambda$. Now, we claim that $\|x_\lambda - z\| \leq \rho$, which implies that $\{x_\lambda\}$ is bounded. Assume to the contrary $\|x_\lambda - z\| > \rho$. The coercivity condition (A$''$) implies that there exists $\bar{x} \in D(A)$ with $\|\bar{x} - z\| < \frac{1}{2}\|x_\lambda - z\|$ such that:

$$0 \leq \langle -\frac{1}{\lambda}(x_\lambda - z), J(x_\lambda - \bar{x})\rangle = \langle -\frac{1}{\lambda}(x_\lambda - \bar{x}), J(x_\lambda - \bar{x})\rangle + \langle -\frac{1}{\lambda}(\bar{x} - z), J(x_\lambda - \bar{x})\rangle$$

Therefore

$$\|x_\lambda - \bar{x}\|^2 \leq \|\bar{x} - z\|\|x_\lambda - \bar{x}\|$$

Since $x_\lambda \neq \bar{x}$, we have: $\|x_\lambda - \bar{x}\| \leq \|\bar{x} - z\|$. Now the triangle inequality yields

$$\|x_\lambda - z\| \leq 2\|\bar{x} - z\|,$$

which is a contradiction. Therefore the sequence $\{x_\lambda\}$ is bounded. It follows that $-\frac{1}{\lambda}(x_\lambda - z) \to 0$ as $\lambda \to +\infty$. Fix $\bar{x} \in D(A)$ and $\bar{y} \in A(\bar{x})$. Since A is accretive we have:

$$\langle -\frac{1}{\lambda}(x_\lambda - z) - \bar{y}, J(x_\lambda - \bar{x})\rangle \geq 0$$

that is

$$0 \leq \langle -\bar{y}, J(x_\lambda - \bar{x})\rangle + \langle -\frac{1}{\lambda}(x_\lambda - z), J(x_\lambda - \bar{x})\rangle \leq \langle -\bar{y}, J(x_\lambda - \bar{x})\rangle + \frac{1}{\lambda}\|x_\lambda - z\|\|x_\lambda - \bar{x}\| \tag{2.1}$$

Letting $\lambda \to \infty$, since $J(x_\lambda - \bar{x})$ is bounded, sequential Banach-Alaoglu theorem implies that exists a sequence $\lambda_n \to \infty$ such that $J(x_{\lambda_n} - \bar{x})$ converges to x^* in weak* topology. Reflexivity of X implies that $x^* \in J(z - \bar{x})$ for some $z \in X$. Now substituting λ by λ_n in (2.1) and letting $\lambda_n \to +\infty$, we have $\langle -\bar{y}, J(z - \bar{x})\rangle \geq 0$. By maximal accretivity of A, we get $z \in A^{-1}(0)$.

If part. Suppose that there exists $x_0 \in D(A)$ satisfying $0 \in A(x_0)$. Since A is accretive, we have $\langle y, J(x - x_0)\rangle \geq 0$, for all $x \in D(A)$ and $y \in A(x)$. Let $\rho = 2\|x_0\| + 1 > 0$ and $z = 0$. Then for every $x \in D(A)$ with $\|x\| > \rho$ and $y \in A(x)$, we have $2\|x_0\| < \|x\|$ and $\langle y, J(x - x_0)\rangle \geqslant 0$. Hence, the coercivity condition (A$''$) holds.

Theorem 2.2 *Let X be reflexive, separable and strictly convex. Suppose $A : X \to 2^X$ is an m-accretive mapping, then the solution set of the inclusion problem* (1.3) *is nonempty and bounded if and only if the coercivity condition* (B$''$) *of Definition 1.1 holds.*

Proof. **If part.** If the coercivity condition (B$''$) holds, then there exist $\rho > 0$ and $z \in X$ such that for each $x \in D(A)$ with $\|x - z\| > \rho$ there is $\bar{x} \in D(A)$ with $\|\bar{x} - z\| < \frac{1}{2}\|x - z\|$ such that

$$\sup_{\bar{y} \in A(\bar{x})} \langle \bar{y}, J(x - \bar{x})\rangle > 0.$$

Since A is accretive, we have:

$$0 \leq \sup_{\bar{y} \in A(\bar{x})} \langle \bar{y}, J(x - \bar{x})\rangle \leq \langle y, J(x - \bar{x})\rangle, \quad \forall y \in A(x).$$

So

$$0 \leq \inf_{y \in A(x)} \langle y, J(x - \bar{x})\rangle.$$

Therefore the coercivity condition (A$''$) holds and by Theorem 2.1, the set $A^{-1}(0)$ is nonempty.

Now we prove the set $A^{-1}(0)$ is bounded. If not, then for any $m > 0$ and $z \in X$ there exists $x_m \in D(A)$ with $\|x_m - z\| > m$ such that $0 \in A(x_m)$. By accretivity of A, we get

$$\langle 0 - y, J(x_m - x)\rangle \geq 0 \quad \forall x \in D(A),\ \forall y \in A(x),$$

therefore

$$\sup_{y \in A(x)} \langle y, J(x_m - x)\rangle \leq 0 \quad \forall x \in D(A). \tag{2.2}$$

By the coercivity condition (B$''$), there exist $\rho > 0$ and $z \in X$ such that for every $x \in D(A)$ with $\|x - z\| > \rho$ there is $x_0 \in D(A)$ with $\|x_0 - z\| < \frac{1}{2}\|x - z\|$ satisfying:

$$\sup_{y_0 \in A(x_0)} \langle y_0, J(x - x_0)\rangle > 0.$$

Taking $m > \rho$ and $x := x_m$, we get

$$\sup_{y_0 \in A(x_0)} \langle y_0, J(x_m - x_0)\rangle > 0,$$

which contradicts (2.2). Hence the set $A^{-1}(0)$ is bounded.

Only if part. Suppose to the contrary, the coercivity condition (B$''$) does not hold. Then for any $\rho > 0$ and $z \in X$ there is $x \in D(A)$ with $\|x - z\| > \rho$ such that for any $\bar{x} \in D(A)$ with $\|\bar{x} - z\| < \frac{1}{2}\|x - z\|$ we have

$$\sup_{\bar{y} \in A(\bar{x})} \langle \bar{y}, J(x - \bar{x}\rangle \leq 0.$$

Let $\rho > 0$ be arbitrary, and $z \in A^{-1}(0)$, which is non-empty by the assumption. Then there is $x \in D(A)$ with $\|x - z\| > \rho$ (by the contrary assumption). Let $\bar{x} = \mathcal{J}_2(x_0) \in$

$D(A)$, where

$$x_0 = \frac{1}{2}z + \frac{1}{2}x. \tag{2.3}$$

We distinguish two cases:

(i) $\bar{x} - z$ and $x_0 - z$ are linearly dependent, i.e. $\bar{x} - z = \lambda(x_0 - z), \quad \lambda \in \mathbb{R}$.

(ii) $\bar{x} - z$ and $x_0 - z$ are linearly independent.

In the case (i), since $\|\bar{x} - z\| = \|\mathcal{J}_2(x_0) - z\| \leq \|x_0 - z\|$, then $-1 \leq \lambda \leq 1$. The case $\lambda = -1$ does not occur. Because it follows that $\mathcal{J}_2(x_0) - z = z - x_0$. Since $\frac{1}{2}(x_0 - \mathcal{J}_2(x_0)) \in A(\mathcal{J}_2(x_0))$ and $0 \in A(z)$, by the accretivity of A, we have:

$$\langle x_0 - \mathcal{J}_2(x_0), J(\mathcal{J}_2(x_0) - z)\rangle \geq 0.$$

By a simple computation, we get: $\mathcal{J}_2(x_0) = z \Rightarrow x_0 = z$. By (2.3), $x = z \in A^{-1}(0)$, which contradicts with $\|x - z\| > \rho > 0$. If $\lambda = 1$, then $\bar{x} = x_0$. In this case $x_0 \in \mathrm{Fix}(\mathcal{J}_2) = A^{-1}(0)$, and $\|x_0 - z\| = \frac{1}{2}\|x - z\| > \frac{\rho}{2}$. This is a contradiction with the boundedness of $A^{-1}(0)$, because ρ is arbitrary. The only remaining case is $-1 < \lambda < 1$. In this case we have:

$$\|\bar{x} - z\| < \|x_0 - z\| = \frac{1}{2}\|x - z\|. \tag{2.4}$$

Now, we show that (2.4) is also concluded from the case (ii) . By the resolvent identity, we have $\mathcal{J}_1(\frac{1}{2}x_0 + \frac{1}{2}\mathcal{J}_2(x_0)) = \mathcal{J}_2(x_0)$, therefore we have

$$\|\bar{x} - z\| = \|\mathcal{J}_2(x_0) - z\| = \|\mathcal{J}_1(\frac{1}{2}x_0 + \frac{1}{2}\mathcal{J}_2(x_0)) - z\|.$$

As $z \in \mathrm{Fix}(\mathcal{J}_1)$ and by (1.1) we obtain

$$\|\bar{x} - z\| \leq \|\frac{1}{2}(x_0 - z) + \frac{1}{2}(\mathcal{J}_2(x_0) - z)\|.$$

From the strict convexity of X and case (ii) it follows that

$$\|\bar{x} - z\| < \frac{1}{2}\|x_0 - z\| + \frac{1}{2}\|\mathcal{J}_2(x_0) - z\| \leq \|x_0 - z\| = \frac{1}{2}\|x - z\|.$$

Hence in both cases, (i) and (ii), we conclude (2.4). Therefore the contrary assumption follows

$$\langle x_0 - \mathcal{J}_2(x_0), J(x - \mathcal{J}_2(x_0))\rangle \leq \sup_{\bar{y} \in A(\bar{x})} \langle \bar{y}, J(x - \bar{x})\rangle \leq 0. \tag{2.5}$$

A simple computation implies that

$$\|x - \mathcal{J}_2(x_0)\|^2 \leq \|x - x_0\|\|x - \mathcal{J}_2(x_0)\|.$$

Since by (2.4), $x \neq \mathcal{J}_2(x_0) = \bar{x}$, from (2.3) we get: $\|x - \bar{x}\| \leq \frac{1}{2}\|x - z\|$. So by the triangle inequality, we obtain

$$\|x - z\| - \|\bar{x} - z\| \leq \|x - \bar{x}\| \leq \frac{1}{2}\|x - z\| \Rightarrow \frac{1}{2}\|x - z\| \leq \|\bar{x} - z\|,$$

which contradicts (2.4).

3. Application to fixed point theory

As discussed in Subsection 1.2, coercivity conditions for an accretive operator can be applied to prove the existence of a fixed point of nonexpansive mappings. The following corollary can be extracted from Subsection 1.2 and Theorem 2.2.
Proof. From the assumption, we have

$$\langle Tx - \bar{x}, J(x - \bar{x})\rangle \leq \|Tx - \bar{x}\|\|x - \bar{x}\| \leq \|x - \bar{x}\|^2,$$

which implies that

$$\langle x - Tx, J(x - \bar{x})\rangle \geq 0.$$

Then the coercivity condition (A$''$) holds for the m-accretive operator $I - T$. Now the result is concluded from Theorem 2.2.

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