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POSITIVE SOLUTIONS OF NONLINEAR THIRD-ORDER BOUNDARY VALUE PROBLEMS INVOLVING STIELTJES INTEGRAL CONDITIONS

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Abstract. In this paper, by using the Guo-Krasnoselskii theorem, we investigate the existence and nonexistence of positive solutions of a class of boundary value problem of third-order nonlinear differential equation involving Stieltjes integral conditions. Under some growth conditions imposed on the nonlinear term, we obtain explicit ranges of values of parameters with which the problem has a positive solution and has no positive solution respectively. An example is given to illustrate the main results of the paper.
Key Words and Phrases: Positive solution, boundary value problem, fixed point, cone.
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References

[1] D.R. Anderson, *Multiple positive solutions for a three-point boundary value problem*, Math. Comput. Modelling, **27**(6)(1998), 49-57.
[2] D.R. Anderson, *Green's function for a third-order generalized right focal problem*, J. Math. Anal. Appl., **288**(2003), 1-14.
[3] J. Chu, Z. Zhou, *Positive solutions for singular non-linear third-order periodic boundary value problems*, Nonlinear Anal., **64**(2006), 1528-1542.
[4] M. El-Shahed, *Positive solutions for nonlinear singular third order boundary value problem*, Commmun. Nonlinear Sci. Numer. Simlat., **16**(2011), 402-413.
[5] J.R. Graef, L. Kong, *Positive solutions for third order semipositone boundary value problems*, Appl. Math. Lett., **22**(2009), 1154-1160.
[6] J. Graef, J.R.L. Webb, *Third order boundary value problems with nonlocal boundary conditions*, Nonlinear Anal., **71**(2009), 1542-1551.
[7] M. Gregus, *Third Order Linear Differential Equations*, in: Math. Appl., Reidel, Dordrecht, 1987.
[8] D. Guo, V. Lakshmikantham, *Nonlinear Problems in Abstract Cones*, Academic Press, New York, 1988.

[9] L. Guo, J. Sun, Y. Zhao, *Existence of positive solutions for nonlinear third-order three-point boundary value problems*, Nonlinear Anal., **68**(2008), 3151-3158.
[10] B. Hopkins, N. Kosmatov, *Third-order boundary value problems with sign-changing solutions*, Nonlinear Anal., **67**(2007), 126-137.
[11] G. Infante, J.R.L. Webb, *Loss of positivity in a nonlinear scalar heat equation*, NoDEA Nonlinear Differential Equations Appl., **13**(2006), 249-261.
[12] T. Jankowski, *Existence of positive solutions to third order differential equations with advanced arguments and nonlocal boundary conditions*, Nonlinear Anal., **75**(2012), 913-923.
[13] R.W. Leggett, L.R. Williams, *Multiple positive fixed points of nonlinear operators on ordered Banach spaces*, Indiana Univ. Math. J., **28**(1979), 673-688.
[14] S. Li, *Positive solutions of nonlinear singular third-order two-point boundary value problem*, J. Math. Anal. Appl., **323**(2006), 413-425.
[15] X. Lin, Z. Du, W. Liu, *Uniqueness and existence results for a third-order nonlinear multi-point boundary value problem*, Appl. Math. Comput., **205**(2008), 187-196.
[16] P. Minghe, S.K. Chang, *Existence and uniqueness of solutions for third-order nonlinear boundary value problems*, J. Math. Anal. Appl., **327**(2007), 23-35.
[17] A.P. Palamides, G. Smyrlis, *Positive solutions to a singular third-order three-point boundary value problem with an indefinitely signed Green's function*, Nonlinear Anal., **68**(2008), 2104-2118.
[18] J. Webb, G. Infante, *Positive solutions of nonlocal boundary value problems: a unified approach*, J. London Math. Soc., **74**(2006), no. 2, 673-693.
[19] J. Webb, K. Lan, *Eigenvalue criteria for existence of multiple positive solutions of nonlinear boundary value problems of local and nonlocal type*, Topol. Methods Nonlinear Anal., **27**(2006), 91-116.
[20] Q. Yao, *Positive solution for a semi-linear third-order two-point boundary value problem*, Appl. Math. Lett., **17**(2004), 1171-1175.

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