

ITERATIVE APPROXIMATIONS OF FIXED POINTS FOR OPERATORS SATISFYING $(B_{\gamma,\mu})$ CONDITION

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Abstract. Let C be a nonempty subset of a Banach space X . A mapping $T : C \rightarrow C$ is said to satisfy $(B_{\gamma,\mu})$ condition if there exists $\gamma \in [0, 1]$ and $\mu \in [0, \frac{1}{2}]$ satisfying $2\mu \leq \gamma$ such that for each $x, y \in C$,

$$\begin{aligned} \gamma \|x - Tx\| &\leq \|x - y\| + \mu \|y - Ty\| \\ \text{implies } \|Tx - Ty\| &\leq (1 - \gamma) \|x - y\| + \mu (\|x - Ty\| + \|y - Tx\|). \end{aligned}$$

In this paper, we obtain some convergence theorems for such mappings using M iterative process in uniformly convex Banach space setting. Our results extend and improve many results in the literature.

Key Words and Phrases: Condition $(B_{\gamma,\mu})$, weak convergence, strong convergence, M iteration, Banach space.

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