***Fixed Point Theory***, 22(2021), No. 2, 795-808
DOI: 10.24193/fpt-ro.2021.2.52
http://www.math.ubbcluj.ro/~nodeacj/sfptcj.html

# SOME VARIANTS OF FIBRE CONTRACTION PRINCIPLE AND APPLICATIONS: FROM EXISTENCE TO THE CONVERGENCE OF SUCCESSIVE APPROXIMATIONS

ADRIAN PETRUŞEL*, IOAN A. RUS** AND MARCEL-ADRIAN ŞERBAN**

*Department of Mathematics, Babeş-Bolyai University, Cluj-Napoca
and
Academy of Romanian Scientists Bucharest, Romania
E-mail: petrusel@math.ubbcluj.ro

**Department of Mathematics, Babeş-Bolyai University, Cluj-Napoca, Romania
E-mail: iarus@math.ubbcluj.ro mserban@math.ubbcluj.ro

**Abstract.** Let $(X_1, \to)$ and $(X_2, \hookrightarrow)$ be two $L$-spaces, $U$ be a nonempty subset of $X_1 \times X_2$ such that $U_{x_1} := \{x_2 \in X_2 \mid (x_1, x_2) \in U\}$ is nonempty, for each $x_1 \in X_1$. Let $T_1 : X_1 \to X_1$, $T_2 : U \to X_2$ be two operators and $T : U \to X_1 \times X_2$ defined by $T(x_1, x_2) := (T_1(x_1), T_2(x_1, x_2))$. If we suppose that $T(U) \subset U$, $F_{T_1} \neq \emptyset$ and $F_{T_2(x_1,\cdot)} \neq \emptyset$ for each $x_1 \in X_1$, the problem is in which additional conditions $T$ is a weakly Picard operator ? In this paper we study this problem in the case when the convergence structures on $X_1$ and $X_2$ are defined by metrics. Some applications to the fixed point equations on spaces of continuous functions are also given.
**Key Words and Phrases**: Triangular operator, fibre contraction, weakly Picard operator, generalized metric space, generalized contraction, well-posedness, Ostrowki property, Ulam-Hyers stability, Volterra operator, functional differential equation, functional integral equation.
**2020 Mathematics Subject Classification**: 47H10, 54H25, 47H09, 45N05, 34K28.

## References

[1] S. András, *Fibre $\varphi-$contraction on generalized metric spaces and applications*, Mathematica, **45**(68)(2003), no. 1, 3-8.
[2] C. Bacoţiu, *Fibre Picard operators on generalized metric spaces*, Sem. Fixed Point Theory Cluj-Napoca, **1**(2000), 5-8.
[3] D.D. Bainov, S.G. Hristova, *Differential Equations with Maxima*, CRC Press, 2011.
[4] L. Barreira, C. Valls, *Parameter dependence of stable manifolds for difference equations*, Nonlinearity, **23**(2010), no. 2, 341-367.
[5] A.M. Bica, S. Mureşan, G. Grebenişan, *Parameter dependence of the solution of second order nonlinear ODE's via Perov's fixed point theorem*, Aust. J. Math. Anal. Appl., **3**(2006), no. 1, Art. 10, 8 pp.
[6] T.A. Burton, *Stability by Fixed Point Theory for Functional Differential Equations*, Dover Publ. New York, 2006.
[7] M.M. Deza, E. Deza, *Encyclopedia of Distances*, Springer, 2009.
[8] M. Dobriţoiu, W.W. Kecs, A. Toma, *An application of the fiber generalized contractions theorem*, WSEAS Trans. Math., **5**(2006), no. 12, 1330-1335.

[9] M. Dobriţoiu, M.-A. Şerban, *Step method for a system of integral equations from biomathematics*, Appl. Math. Comput., **227**(2014), 412-421.

[10] W.M. Hirsch, C.C. Pugh, *Stable manofolds and hyperbolic sets*, Proc. Symp. in Pure Math., **14**(1970), 133-163.

[11] V. Ilea, D. Otrocol, I.A. Rus, M.A. Şerban, *Applications of fibre contraction principle to some classes of functional integral equations*, to appear.

[12] V. Ilea, D. Otrocol, M.-A. Şerban, D. Trif, *Integro-differential equations with two times lags*, Fixed Point Theory, **13**(2012), no. 1, 85-97.

[13] V. Mureşan, *Existence, uniqueness and data dependence for the solution of a Fredholm integral equation with linear modification of the argument*, Acta Sci. Math. (Szeged), **68**(2002), no. 1-2, 117-124.

[14] V. Mureşan, *Existence, uniqueness and data dependence results for the solution of a Volterra-Sobolev functional-integral equation*, Automat. Comput. Appl. Math. **19**(2010), no. 2, 343-350.

[15] V. Mureşan, *On a functional-differential equation*, in: Fixed Point Theory and its Applications, (Eds. R. Espínola, A. Petruşel and S. Prus), House of the Book of Science, Cluj-Napoca, 2013, 201-207 pp.

[16] I.M. Olaru, *An integral equation related to some epidemic model*, Automat. Comput. Appl. Math., **19**(2010), no. 2, 367-374.

[17] D. Otrocol, *Smooth dependence on parameters for some Lotka-Volterra system with delays*, An. Univ. Vest Timiş. Ser. Mat.-Inform., **43**(2005), no. 1, 109-114.

[18] D. Otrocol, M.A. Şerban, *An efficient step method for a system of differential equations with delay*, J. Appl. Anal. Comput., **8**(2018), no. 2, 498-508.

[19] A. Petruşel, G. Petruşel, *A study of a general system of operator equations in b-metric spaces via the vector approach in fixed point theory*, J. Fixed Point Theory Appl., **19**(2017), 1793-1814.

[20] A. Petruşel, G. Petruşel, *Coupled fixed points and coupled coincidence points via fixed point theory*, in: Mathematical Analysis and Applications, Book Editors: M. Ruzhansky, H. Dutta, R.P. Agarwal, Wiley, 2018, pp. 661-717.

[21] A. Petruşel, I.A. Rus, *Fixed point equations with abstract Volterra operators on spaces of functions of several variables*, to appear.

[22] A. Petruşel, I.A. Rus, M.A. Şerban, *Fixed points for operators on generalized metric spaces*, CUBO A Mathematical Journal, **10**(2008), no. 4, 45-66.

[23] I.A. Rus, *A fibre generalized contraction theorem and applications*, Mathematica, **41**(1999), no. 1, 85-90.

[24] I.A. Rus, *Fibre Picard operators and applications*, Stud. Univ. Babeş-Bolyai Math., **44**(1999), 89-98.

[25] I.A. Rus, *Fibre Picard operators on generalized metric spaces and applications*, Scripta Sc. Math., **1**(1999), 326-334.

[26] I.A. Rus, *Generalized Contractions and Applications*, Cluj University Press Cluj-Napoca, 2001.

[27] I.A. Rus, *Picard operators and applications*, Sci. Math. Japonica, **58**(2003), 191-219.

[28] I.A. Rus, *Abstract models of step method which imply the convergence of successive approximations*, Fixed Point Theory, **9**(2008), 293-307.

[29] I.A. Rus, *Some nonlinear functional differential and integral equations via weakly Picard operator theory: a survey*, Carpathian J. Math., **26**(2010), 230-258.

[30] I.A. Rus, *Some variants of contraction principle, generalizations and applications*, Stud. Univ. Babeş-Bolyai Math., **61**(2016), no. 3, 343-358.

[31] I.A. Rus, *Some variants of contraction principle in the case of operators with Volterra property: step by step contraction principle*, Adv. Theory of Nonlinear Anal. Appl., **3**(2019), 111-120.

[32] I.A. Rus, A. Petruşel, G. Petruşel, *Fixed Point Theory*, Cluj Univ. Press, Cluj-Napoca, 2008.

[33] I.A. Rus, A. Petruşel, M.A. Şerban, *Fibre Picard operators on gauge spaces and applications*, Zeitschrift für Analysis und ihre Anwendungen, **27**(2008), 399-415.

[34] I.A. Rus, M.A. Şerban, *Operators on infinite dimensional Cartesian product*, An. Univ. Vest Timiş. Ser. Mat.-Inform., **48**(2010), no. 1-2, 253-263.

[35] I.A. Rus, M.A. Şerban, *Some generalizations of a Cauchy lemma and applications*, Topics in Mathematics, Computer Science and Philosophy, Editor Şt. Cobzaş, Cluj University Press, 2008, 173-181.

[36] I.A. Rus, M.A. Şerban, D. Trif, *Step method for some integral equations from biomathematics*, Bull. Math. Soc. Sci. Math. Roumanie, **54**(102)(2011), no. 2, 167-183.

[37] J. Sotomayor, *Smooth dependence of solutions of differential equations on initial data: a simple proof*, Bol. Soc. Bras. Mat., **4**(1973), 55-59.

[38] M.A. Şerban, *Application of fiber Picard operators to integral equations*, Bul. Ştiinţ. Univ. Baia Mare, Ser. B, Matematică-Informatică, **43**(2002), no. 1, 119-128.

[39] M.A. Şerban, *Fibre contraction theorem in generalized metric spaces*, Automat. Comput. Appl. Math., **16**(2007), no. 1, 139-144.

[40] M.A. Şerban, *Fibre $\varphi-$contractions*, Stud. Univ. Babeş-Bolyai Math., **44**(1999), no. 3, 99-108.

[41] M.A. Şerban, *Saturated fibre contraction principle*, Fixed Point Theory, **18**(2017), no. 2, 729–740.

[42] M.A. Şerban, *Fixed Point Theory for Operators on Cartesian Product*, (Romanian), Cluj University Press, Cluj-Napoca, 2002.

[43] A. Tămăşan, *Differentiability with respect to lag for nonlinear pantograph equations*. Modern applied analysis (Ilieni, 1997), Pure Math. Appl., **9**(1998), no. 1-2.

[44] A. Vanderbauwhede, *Centre manifolds, normal forms and elementary bifurcations*, Dynamics Reports, Vol. 2, 89–169, Dynam. Report. Ser. Dynam. Systems Appl., 2, Wiley, Chichester, 1989.

*Received: January 7, 2020; Accepted: December 14, 2020.*