

## DECAY ESTIMATES FOR TWO-TERM TIME FRACTIONAL DIFFERENTIAL EQUATIONS WITH INFINITE DELAYS

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**Abstract.** In this paper, nonlinear differential evolution equations of fractional order in Banach spaces involving unbounded delays are investigated. We aim to prove the existence of mild solutions and demonstrate its polynomial decay by the fixed point principle for condensing maps. An example of the application of abstract results is given for illustration.

**Key Words and Phrases:** Fractional differential equations, functional differential equations, decay estimates of mild solutions, measure of noncompactness.

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