

## EXISTENCE AND APPROXIMATION OF FIXED POINTS OF $\lambda$ -HYBRID MAPPINGS IN COMPLETE CAT(0) SPACES

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**Abstract.** In this paper, we first state and establish some general existence theorems in Hadamard spaces. Then, we introduce class of  $\lambda$ -hybrid mappings in Hadamard spaces and prove existence of fixed point for such mappings. Finally, we establish convergence theorem of Mann's iterative process for  $\lambda$ -hybrid mappings.

**Key Words and Phrases:** Existence theorem,  $\lambda$ -hybrid mappings, nonexpansive mapping, fixed point, CAT(0) metric space.

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### REFERENCES

- [1] S. Alizadeha, H. Dehghan, F. Moradlou,  $\Delta$ -convergence theorems for inverse-strongly monotone mappings in CAT(0) spaces, *Fixed Point Theory*, **19**(2018), no. 1, 45-56.
- [2] K. Aoyama, S. Iemoto, F. Kohsaka, W. Takahashi, *Fixed point and ergodic theorems for  $\lambda$ -hybrid mappings in Hilbert spaces*, *J. Nonlinear Convex Anal.*, **11**(2010), 335-343.
- [3] M. Bacak, *Convex Analysis and Optimization in Hadamard Spaces*, De Gruyter, Berlin, 2014.
- [4] W. Ballmann, *Lectures on Spaces of Nonpositive Curvature*, in: DMV Seminar Band, vol. 25, Birkhäuser, Basel, 1995.
- [5] I.D. Berg, I.G. Nikolaev, *Quasilinearization and curvature of Alexandrov spaces*, *Geom. Dedicata*, **133**(2008), 195-218.
- [6] E. Blum, W. Oettli, *From optimization and variational inequalities to equilibrium problems*, *Math. Student*, **63**(1994), 123-145.
- [7] M. Bridson, A. Haefliger, *Metric Spaces of Nonpositive Curvature*, Springer-Verlag, Berlin, Heidelberg, New York, 1999.
- [8] H. Dehghan, J. Rooin, *A characterization of metric projection in CAT(0) spaces*, arXiv:1311.4174v1, 2013.
- [9] S. Dhompongsa, W.A. Kirk, B. Panyanak, *Nonexpansive set-valued mappings in metric and Banach spaces*, *J. Nonlinear and Convex Anal.*, **8**(2007), 35-45.
- [10] S. Dhompongsa, W.A. Kirk, B. Sims, *Fixed points of uniformly lipschitzian mappings*, *Nonlinear Anal.*, **65**(2006), 762-772.

- [11] S. Dhompongsa, B. Panyanak, *On  $\Delta$ -convergence theorems in  $CAT(0)$  spaces*, Comput. Math. Appl., **56**(2008), 2572-2579.
- [12] A. Gharajelo, H. Dehghan, *Convergence theorems for strict pseudo-contractions in  $CAT(0)$  metric spaces*, Filomat, **31**(2017), 1967-1971.
- [13] B.A. Kakavandi, *Weak topologies in complete  $CAT(0)$  metric spaces*, Proc. Amer. Math. Soc., 0002-9939 (2012), 11743-5.
- [14] W.A. Kirk, *Some recent results in metric fixed point theory*, J. Fixed Point Theory Appl., **2**(2007), 195-207.
- [15] W.A. Kirk, B. Panyanak, *A concept of convergence in geodesic spaces*, Nonlinear Anal., **68**(2008), 3689-3696.
- [16] F. Kohsaka, W. Takahashi, *Fixed point theorems for a class of nonlinear mappings related to maximal monotone operators in Banach spaces*, Arch. Math., Basel, **91**(2008), 166-177.
- [17] T.C. Lim, *Remarks on some fixed point theorems*, Proc. Amer. Math. Soc., **60**(1976), 179-182.
- [18] A. Papadopoulos, *Metric Spaces, Convexity and Nonpositive Curvature*, IRMA Lectures in Mathematics and Theoretical Physics, 6. European Mathematical Society (EMS), Zürich, 2005.
- [19] A. Pazy, *Asymptotic behavior of contractions in Hilbert space*, Israel J. Math., **9**(1971), 230-240.
- [20] S. Ranjbar, H. Khatibzadeh, *Strong and  $\Delta$ -convergence to a zero of a monotone operator in  $CAT(0)$  spaces*, Mediterr. J. Math., (2017), 14:56.
- [21] H.L. Royden, *Real Analysis*, Macmillan, 1963.
- [22] W. Takahashi, *Existence theorems generalizing fixed point theorems for multivalued mappings*, In: M.A. Thera, J.B. Baillon (Eds.), Fixed Point Theory and Applications, Pitman Research Notes in Mathematics Series, 252, Wiley, New York, 1991, 397-406.
- [23] W. Takahashi, *Fixed point theorems for new nonlinear mappings in a Hilbert space*, J. Nonlinear Convex Anal., **11**(2010), 79-88.
- [24] W. Takahashi, *Nonlinear Functional Analysis*, Yokohoma Publishers, Yokohoma, 2000.
- [25] W. Takahashi, J.C. Yao, *Fixed point theorems and ergodic theorems for nonlinear mappings in a Hilbert space*, Taiwanese J. Math., **15**(2011), 457-472.
- [26] R. Wangkeeree, P. Preechasilp,  *$\Delta$ -convergence for generalized hybrid mappings in  $CAT(0)$  spaces*, Bull. Malays. Math. Sci. Soc., **38**(2015), 1127-1141.

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