

APPROXIMATING COMMON FIXED POINT VIA ISHIKAWA'S ITERATION

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Abstract. In this work, we approximate a common fixed point of mappings $F, G : M \cup N \rightarrow M \cup N$, satisfying the conditions

- (1) $G(M) \subseteq M, G(N) \subseteq N, F(M) \subseteq M$ and $F(N) \subseteq N$;
- (2) $\|Fu - Gv\| \leq \|u - v\|$ for $u \in M, v \in N$; and
- (3) $\|Fu - Gv\| \leq \|u - v\|$ for $u \in N, v \in M$,

where M and N are nonempty bounded closed convex subsets of a uniformly convex Banach space. We consider Ishikawa iteration associated with F and G and von Neumann sequence associated with Ishikawa iteration to approximate the common fixed point of F and G . We prove convergent results for common fixed point of F and G . Finally, we give corollaries on common best proximity point for cyclic mappings.

Key Words and Phrases: Nonexpansive mappings, best proximity points, fixed points, Banach space, Von Neumann sequences.

2020 Mathematics Subject Classification: 47H10, 46B20, 54H25.

Acknowledgements. The authors would like to thank the National Board for Higher Mathematics (NBHM), DAE, Govt. of India for providing a financial support under the grant no. 02011/22/2017/R&D II/14080.

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Received: October 3, 2019; Accepted: May 16, 2020.