

FIXED POINTS AND TOPOLOGICAL PROPERTIES OF EXTENDED QUASI-METRIC SPACES

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Abstract. In this paper, we introduce an extension of metric spaces, which includes S spaces. Some properties of this topological structure are analyzed. Also, a non-compactness measure and condensing correspondences are defined for these type of spaces and natural results are obtained. Moreover, fixed point theorems for functions and correspondences satisfying certain Banach orbital condition are introduced and proved. These results are applied to contractions, which are defined by means of the extended quasi-metric, both for functions and correspondences.

Key Words and Phrases: Quasi-metric spaces, non-compactness measure, fixed point, contractions, correspondences, lower and upper semicontinuity, b -metric spaces.

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