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ON THE BURTON METHOD OF PROGRESSIVE CONTRACTIONS FOR VOLTERRA INTEGRAL EQUATIONS

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Abstract. In the paper [4] the author give a new method to study the existence and uniqueness of a solution on $[0,\infty[$ of a scalar integral equation

$$x(t) = g(t, x(t)) + \int_0^t A(t-s)f(t, s, x(s))ds,$$

where $u, v \in \mathbb{R},\ t \in [0,\infty[$ imply that there exists $0 < l < 1$ with

$$|g(t,u) - g(t,v)| \leq l\,|u-v|$$

and for each $b > 0$ there exists $L_b > 0$ such that

$$|f(t,u) - f(t,v)| \leq L_b\,|u-v|\,,\ \forall t \in [0,b],\ \forall u,v \in \mathbb{R}.$$

In this paper we extend the Burton method to the case where instead of scalar equations we consider an equation in a Banach space.
Key Words and Phrases: Progressive contractions, fixed points, existence, uniqueness, integro-differential equations.
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