FIXED POINT PROPERTIES RELATED TO CHARACTER AMENABLE BANACH ALGEBRAS

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Abstract. For a Banach algebra $\mathfrak{A}$, we investigate several fixed point properties for $\mathfrak{A}$ with respect to a non-zero character ϕ on $\mathfrak{A}$. As the main results, we obtain some fixed point characterizations for ϕ-amenability of $\mathfrak{A}$. We also describe ϕ-amenability of $\mathfrak{A}$ in terms of the Hahn-Banach extension property. As a consequence, we offer some applications to the group algebra and the Fourier algebra of a locally compact group.

Key Words and Phrases: Banach algebra, character amenability, fixed point property, the Hahn-Banach extension property, locally compact group.

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