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ADDENDUM TO THE PAPER "AN ITERATIVE METHOD FOR A FUNCTIONAL-DIFFERENTIAL EQUATION OF SECOND ORDER WITH MIXED TYPE ARGUMENT", FIXED POINT THEORY, 14(2013), NO. 2, 427-434

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In the paper "An iterative method for a functional-differential equation of second order with mixed type argument", Fixed Point Theory, 14(2013), No. 2, 427-434, we study the problem

$$x''(t) = f(t, x(t), x'(t), x(t-h), x(t+h)),\ t \in [-h, T], \tag{1}$$

$$x(t) = \varphi(t),\ t \in [-h, h] \tag{2}$$

in the following conditions

(C$_1$) $f \in C^k([-T,T] \times \mathbb{R}^4, \mathbb{R}), \varphi \in C^k([-h,h], \mathbb{R}), k = \left[\dfrac{T}{h}\right] + 1;$

(C$_2$) $\dfrac{\partial f(t,u,v,w,z)}{\partial z} \in \mathbb{R}^*,\ \forall t \in [-T,T],\ \forall u,v,w,z \in \mathbb{R};$

(C$_3$) $\left|\dfrac{\partial f(t,u,v,w,z)}{\partial z}\right| \le M_1, \forall t \in [-T,T],\ \forall u,v,w,z \in \mathbb{R};$

(C$_4$) $\forall t \in [-T,T], u,v,w,z,\eta \in \mathbb{R}$, the equation $f(t,u,v,w,z) - \eta = 0$ has a unique solution.

In this addendum we add the condition

(C_5) $\dfrac{d^k}{dt^k}\varphi''(t)\Big|_{t=0} = \frac{d^k}{dt^k} f\Big(t, \varphi(t), \varphi'(t), \varphi(t-h), \varphi(t+h)\Big)\Big|_{t=0}.$

So the final form of Theorem 2.1 should be:

Theorem 2.1. *In the conditions* $(C_1) - (C_3)$ *we have*

a) The problem (1.1)-(1.2) has in $C^2[-T,T]$ *(which is in fact in* $C^k[-T,T]$*) a unique solution*

$$x^*(t) = \begin{cases} \varphi(t), & t \in [-h,h] \\ x_1^*(t), & t \in [h,2h] \\ \vdots & \\ x_n^*(t), & t \in [nh,T]. \end{cases}$$

b) We suppose that the conditions $(C_1) - (C_5)$ *are satisfied. Then the sequence defined by*

(p_0) $x(t) = \varphi(t) = \begin{cases} x_{-1}(t), t \in [-h,0], \\ x_0(t), t \in [0,h]; \end{cases}$

(p_1) $x_{1m}(t) = x_{1,m-1}(t) - G(t, x_1^*(t))F(t, x_{1,m-1}(t)), t \in [h,2h];$

(p_2) $x_{2m}(t) = x_{2,m-1}(t) - G(t, x_2^*(t))F(t, x_{2,m-1}(t)), t \in [2h,3h];$

(p_3) $x_{3m}(t) = x_{3,m-1}(t) - G(t, x_3^*(t))F(t, x_{3,m-1}(t)), t \in [3h,4h];$

$\vdots$

(p_n) $x_{nm}(t) = x_{n,m-1}(t) - G(t, x_n^*(t))F(t, x_{n,m-1}(t)), t \in [nh,T].$

is convergent and $\lim\limits_{m\to\infty} x_{im} = x_i^*,\ i = \overline{1,n};$

Proof. For the initial proof we add the following: on each interval of the form: $[kh,(k+1)h] \cup [(k+1)h,T]$, $k \in \mathbb{Z}$, from condition $(C5)$ we have $x_k(kh) = x_{k-1}(kh)$, $x_k^{'}(kh) = x_{k-1}^{'}(kh)$ and $x_k^{''}(kh) = x_{k-1}^{''}(kh)$. We choose a start function $x_{k,0}(t)$ such that $x_{k,0}(kh) = x_{k-1,0}(kh)$. So we obtain $x_{k,m}(kh) = x_{k-1,m}(kh)$.

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