

J^* -HOMOMORPHISMS AND J^* -DERIVATIONS ON J^* -ALGEBRAS FOR A GENERALIZED JENSEN TYPE FUNCTIONAL EQUATION

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Abstract. We will apply the fixed point method for proving the stability and superstability of J^* -homomorphisms and J^* -derivations associated to a generalized Jensen type functional equation between J^* -algebras.

Key Words and Phrases: Approximate J^* -homomorphism; approximate J^* -derivation; J^* -algebra; alternative fixed point; generalized Jensen functional equation.

2010 Mathematics Subject Classification: 46L57, 16W25, 39B82, 47B47, 47H10.

Acknowledgement. The authors would like to extend their thanks to referee for his (her) valuable comments and suggestions which helped simplify and improve the results of paper.

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Received: July 6, 2010; Accepted: October 12, 2011.