

## Laborator 2: Variational Calculus. First and second order variation of a functional

1. Calculate the following integrals:

(a)  $\int \cos^2(x) dx$

(b)  $\int_0^\pi \sin(x) dx$

(c)  $\int \frac{x}{x^3 - 1} dx$

(d)  $\int_1^2 \frac{\sin^2(x)}{x} dx$

(e)  $\int_1^4 e^{x^2} x dx$

2. Let consider the following integral functionals:

(a)  $I[y] = \int_1^2 \frac{[y'(x)]^2}{x} dx$

(b)  $I[y] = \int_1^2 \frac{\sqrt{1 + y'(x)^2}}{x} dx$

(c)  $I[y] = \int_1^2 \frac{[y'(x)]^2}{x^3} dx$

(d)  $I[y] = \int_1^2 [(y'(x))^2 - 8xy' + x] dx$

Evaluate these functionals for the following functions:

$$y(x) = (3x^3 + 5x^2)/x + 1, \quad y(x) = \sin(x), \quad y(x) = e^x, \quad y(x) = x^2$$

3. Calculate first and second order variation for the following functionals:

(a)  $I[y] = \int_0^1 y(x)^3 y'(x) dx$

(b)  $I[y] = \int_{-1}^1 [y(x) + y(x)^2] y'(x) dx$

(c)  $I[y] = \int_0^1 [y(x) + xy'(x)^2] dx$

(d)  $I[y] = \int_1^2 y(x)^2 y'(x)^2 dx$