Laboratory 1: Solving differential equations with MAPLE

1. Find the general solution of the differential equations and draw the graph for some solutions:

   (a) $2x^2y' = x^2 + y^2$
   (b) $y' = -\frac{x+y}{y}$
   (c) $y'' + y = \sin x + \cos x$
   (d) $y'' - y = e^{2x}$

2. Solve the following IVPs and draw the solution graph:

   (a) $y' = 1 + y^2 \, , \, y(0) = 1;$
   (b) $y' = \frac{1}{1-x^2}y + 1 + x \, , \, y(0) = 0;$
   (c) $y'' - 5y' + 4y = 0 \, , \, y(0) = 5, \, y'(0) = 8;$
   (d) $y'' - 4y' + 5y = 2x^2e^x \, , \, y(0) = 2, \, y'(0) = 3;$

3. Check if the given functions are solution for the specified differential equation:

   (a) $y' = 1 + x \, , \, y(x) = x^2/2 + x;$
   (b) $y' = y + 1 \, , \, y(x) = 2e^x - 1;$
   (c) $y'' - 5y' + 4y = 0 \, , \, y(x) = \sin(x);$
   (d) $y'' - 2y' + y = 2x^2e^x \, , \, y(x) = e^{2x};$

4. Let $\alpha \in \mathbb{R}$ and $x(\cdot, \alpha)$ the solution of the IVP

   $$x'' - 4x = \alpha t, \quad x(0) = \alpha, \quad x'(0) = 0.$$

   Prove that $\lim_{\alpha \to 0} x(t, \alpha) = 0$ for any $t \in \mathbb{R}$.

5. Let $\omega > 0$ and $x(\cdot, \omega)$ the solution of the IVP

   $$x'' + x = \cos(\omega t), \quad x(0) = x'(0) = 0.$$

   Prove that $\lim_{\omega \to 1} x(t, \omega) = x(t, 1)$ for any $t \in \mathbb{R}$.

6. Find the solution of the IVP and draw the corresponding graph

   $$x'' + x = f(t), \quad x(0) = 5, \quad x'(0) = 0,$$

   unde $f(t) = \begin{cases} t, & t \in [0, \pi/2) \\ \pi - t, & t \in [\pi/2, \pi] \\ 0, & t \in (\pi, \infty) \end{cases}$.

   Hint: to define picewise function use **picewice** command. (see MAPLE help).