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EXTENDED RESULTS ON MULTIPLICATION Γ-SEMIGROUPS

JOHNSON ADEREMI AWOLOLA, MUSA ADEKU IBRAHIM, and ABDUSSAMAD TANKO IMAM

Abstract. The theory of Γ-semigroups is an extension of the semigroup theory. In this paper, we have studied some new results on right multiplication Γ-semigroups. Particularly, the condition for a principal right ideal Γ-semigroup to be a right multiplication Γ-semigroup and the construction of larger right multiplication Γ-semigroups by considering the direct product.

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1. INTRODUCTION

Semigroups are algebraic structures that have remained a very lively area of research within the mathematical community. The formal investigation of semigroups was initiated at the beginning of the 20th century. Before this, however, much groundwork was laid by researchers, as seen in [8, 11], arriving at the study of semigroups from the direction of group and ring theory. The earliest contributions to the theory of semigroups were strongly motivated by comparisons with groups and rings, which significantly promoted the study of language and coding theory, combinatorics, automata, and mathematical analysis [10].

The groundbreaking work of [16] introduced a new kind of algebraic structure termed the Γ-ring, representing a significant extension of the classical ring theory. This seminal idea has catalysed a surge of research activity among numerous scholars, resulting in a substantial body of new findings, as evidenced by the notable contributions in [5, 6, 13, 14, 17]. Inspired by the profound implications of this ring generalisation, a parallel exploration unfolds within the domain of semigroup theory. The paper [20] spearheaded this endeavour by pioneering the theory of Γ-semigroups. Subsequently, [21] collaborated, undertook a refinement of the defining conditions, slightly adjusting them to redefine Γ-semigroups, and further advanced the theoretical framework.

The development of Γ-semigroups hinges on the fact that subsets of a semigroup inherently exhibit associativity but are not necessarily closed. This

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Corresponding author: Johnson Aderemi Awolola.

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fundamental insight has paved the way for numerous generalisations and analogues of corresponding results in semigroup theory stemming from the modified definition in [2, 3, 4, 7, 9, 12, 18, 22, 23, 24].

Investigations on the extension of semigroup structures to Γ-semigroups have largely been studied, as previously mentioned, including multiplication semigroups. Awolola and Ibrahim [1] introduced and established some properties of multiplication Γ-semigroup. The motivation behind their study is based on the two-fold concept of multiplication semigroups presented by [15]. In this paper, the notion of right multiplication Γ-semigroups is broadened by investigating some new results and, in particular, the condition for a principal right ideal Γ-semigroup to be a right multiplication Γ-semigroup and the construction of larger right multiplication Γ-semigroups by taking the direct product into consideration.

2. PRELIMINARIES

We review some of the fundamental terminology and results in the theory of Γ-semigroups as concerned in this paper from [21] and [1], respectively.

Definition 2.1. Let $S = \{a, b, c...\}$ and $\Gamma = \{\alpha, \beta, \gamma, ...\}$ be two non-empty sets. Then S is called a Γ-*semigroup* if there exists a mapping $S \times \Gamma \times S \longrightarrow S \mid (a, \alpha, b) \longrightarrow a\alpha b \in S$ satisfying $(a\alpha b)\beta c = a\alpha(b\beta c)$ for all $a, b, c \in S$ and $\alpha, \beta \in \Gamma$.

Let S be an arbitrary semigroup and Γ any non-empty set. Define $S \times \Gamma \times S \longrightarrow S$ by $a\alpha b = ab$ for all $a, b \in S$ and $\alpha \in \Gamma$. Then S is a Γ-semigroup. Thus, a semigroup can be considered to be a Γ-semigroup. However, any Γ-semigroup need not be a semigroup. For instance, let $S = \{-\mathrm{i}, \mathrm{i}, 0\}$ and $\Gamma = S$. Then S is a Γ-semigroup with respect to multiplication of complex numbers, whereas S does not reduce to a semigroup with respect to multiplication of complex numbers.

A non-empty subset $A \subseteq S$ is called a Γ-*subsemigroup* if

$$A\Gamma A = \{a\gamma b \mid a, b \in A,\ \gamma \in \Gamma\} \subseteq A.$$

If A and B are two non-empty subsets of a Γ-semigroup S, then $A\Gamma B$ is defined as $A\Gamma B = \{a\gamma b \mid a \in A,\ b \in B \text{ and } \gamma \in \Gamma\}$. For simplicity we write $a\Gamma B$, $A\Gamma b$ and $A\gamma B$ in place of $\{a\}\Gamma B$, $A\Gamma\{b\}$ and $A\{\gamma\}B$ respectively.

Definition 2.2. Let S be a Γ-semigroup. A non-empty subset A of S is called a *left (right)* Γ-*ideal* (or simply a left (right) ideal) of S if $S\Gamma A \subseteq A$ ($A\Gamma S \subseteq A$). Further, a non-empty subset A of a Γ-semigroup S is called a Γ-*ideal* if A is both a left and a right Γ-ideal of S.

A (left, right) Γ-ideal A is an *idempotent* (left, right) Γ-ideal if $A = A\Gamma A$.

Definition 2.3. A Γ-semigroup S is called *left (right) simple* if it contains no proper left (right) Γ-ideal, i.e., for every $a \in S$, $S\Gamma a = S$ ($a\Gamma S = S$). A Γ-semigroup S is said to be *semisimple* if $\langle a\rangle \Gamma \langle a\rangle = \langle a\rangle$ for each element a of S.

Definition 2.4. A Γ-semigroup S is called *regular* if for each $a \in S$, there exist $x \in S$ and $\alpha, \beta \in \Gamma$ such that $a = a\alpha x\beta a$. Now, S is termed *left (right) regular* if for each $a \in S$, there exist $x \in S$ and $\alpha, \beta, \gamma \in \Gamma$ such that $a = x\alpha a\beta a$ $(a = a\alpha a\beta x)$.

Definition 2.5. Let S be a Γ-semigroup. An element a of a Γ-semigroup S is called an *α-idempotent* if $a\alpha a = a$ for some $\alpha \in \Gamma$. The set of all α-idempotents is denoted by E_α and we denote $\bigcup_{\alpha\in\Gamma} E_\alpha$ by $E(S)$. The elements of $E(S)$ are called *idempotent elements of* S. Every element of $E(S)$ is called an idempotent element of S. In a regular Γ-semigroup S, we have that $E(S)$ is a non-empty set.

Definition 2.6. A Γ-semigroup S is said to be *left (right) cancellative* if $a\alpha x = a\alpha y \Rightarrow x = y$ $(x\beta b = y\beta b \Rightarrow x = y)$ for all $a, b, x, y \in S,\ \alpha, \beta \in \Gamma$. A Γ-semigroup S is called *cancellative* if S is both left and right Γ-cancellative.

It is well known that for each element a of a Γ-semigroup S, the left Γ-ideal $a \cup S\Gamma a$ containing a is the smallest left Γ-ideal of S containing a. If A is any other left Γ-ideal containing a, then $a \cup S\Gamma a \subseteq A$. This is denoted by $\langle a\rangle_l$ and called the *principal left Γ-ideal generated by the element* a. Similarly, for each $a \in S$, the smallest right Γ-ideal containing a is $a \cup a\Gamma S$, which is denoted by $\langle a\rangle_r$ and called the *principal right Γ-ideal generated by the element* a. The principal Γ-ideal of S generated by the element a is denoted by $\langle a\rangle$ and $\langle a\rangle = a \cup S\Gamma a \cup a\Gamma S \cup S\Gamma a\Gamma S$.

Now, the concept of right multiplication Γ-semigroup in a non-commutative case is presented as follows.

Definition 2.7. Let S be a non-commutative Γ-semigroup. Then S is called a *right multiplication Γ-semigroup* if for any two right Γ-ideals A, B of S such that $A \subseteq B$, there exists a right Γ-ideal C of S such that $A = C\Gamma B$. Dually, *left multiplication Γ-semigroups* can be defined.

The example below is constructed to bring the idea into focus.

Example 2.8. Let $S = \{1, -1, \mathrm{i}, -\mathrm{i}\}$ be a semigroup under the operation given by the table below:

$\cdot$	1	-1	i	$-\mathrm{i}$
1	1	1	1	1
-1	1	-1	-1	1
i	1	i	i	1
$-\mathrm{i}$	$-\mathrm{i}$	$-\mathrm{i}$	$-\mathrm{i}$	$-\mathrm{i}$

Let $\Gamma = \{\alpha\}$. Define $a\alpha b = ab$. Since $a\alpha(b\alpha c) = (a\alpha b)\alpha c$ for all $a, b, c \in S$, S is a Γ-semigroup. Let $A = \{1\}$, $B = \{1, -i\}$ and $C = \{1, i\}$. Then A, B and C are all right Γ-ideals of the Γ-semigroup S. Since $A = C\Gamma B$, S is a right multiplication Γ-semigroup.

The definition leads to the following remark.

REMARK 2.9. In a right multiplication Γ-semigroup S, $A = A\Gamma B$ for every right Γ-ideal A of S.

PROPOSITION 2.10. *Let S be a right multiplication Γ-semigroup. Then:*
(i) $S\Gamma S = S$.
(ii) $x \in x\Gamma S$ *for each* $x \in S$.

PROPOSITION 2.11. *Let S be a right multiplication Γ-semigroup. Then $A \subseteq S\Gamma A$ for any right Γ-ideal.*

PROPOSITION 2.12. *Let S be a Γ-semigroup. If $a \in (a\Gamma S)\Gamma(a\Gamma S)$ for every $a \in S$, then S is a right multiplication Γ-semigroup.*

PROPOSITION 2.13. *Let S be a Γ-semigroup. If S is regular (right regular), then S is a right multiplication Γ-semigroup.*

PROPOSITION 2.14. *Let S be a right multiplication Γ-semigroup. If S contains a left canellative element, then S contains a β-idempotent which is a left identity.*

THEOREM 2.15. *Suppose that S is a Γ-semigroup such that every right Γ-ideal is a Γ-ideal. Then S is a semisimple right multiplication Γ-semigroup if and only if S is right regular.*

THEOREM 2.16. *Let S be a left cancellative Γ-semigroup. If S is semisimple right multiplication Γ-semigroup, then S is a simple Γ-semigroup such that $a \in a\Gamma S$ for each $a \in S$.*

3. SOME NEW RESULTS ON MULTIPLICATION Γ-SEMIGROUPS

THEOREM 3.1. *Let S be a right multiplication Γ-semigroup. If there exists an idempotent right Γ-ideal B of S that is also a left Γ-ideal, then B is a right multiplication Γ-semigroup.*

Proof. Let A be a right Γ-ideal of B. Then $A\Gamma S = C\Gamma B$ for some right Γ-ideal C of S since $A\Gamma S \subseteq B\Gamma S = B = S\Gamma B$. Now,

$$A\Gamma S = C\Gamma B = C\Gamma B\Gamma B = A\Gamma S\Gamma B = A\Gamma B \subseteq A$$

and thus B is a right Γ-ideal of S. Since $A \subseteq B$ and A, B are right Γ-ideals of S so that $A = D\Gamma B$ for some right Γ-ideal D of S. Therefore,

$$A = D\Gamma B = D\Gamma(B\Gamma B) = A\Gamma B.$$

Now, for any right Γ-ideal A of B we have $A = A\Gamma B$. Let A_1 and A_2 be any two right Γ-ideals of B such that $A_1 \subseteq A_2$. Then A_1 and A_2 are also right Γ-ideals of S so that $A_1 = R\Gamma A_2$ for some right Γ-ideal R of S. Since $A_2 = A_2\Gamma B$ we have $A_1 = R\Gamma(A_2\Gamma B)$. Now, $A_2\Gamma B$ is a right Γ-ideal of B. Hence, B is a right multiplication Γ-semigroup. $\square$

THEOREM 3.2. *Let S be a right multiplication Γ-semigroup. If every left Γ-ideal is Γ-ideal, then S is left regular.*

Proof. Suppose that the left Γ-ideal $a \cup S\Gamma a$ is a Γ-ideal of S. Then we have $a\Gamma S \subseteq a \cup S\Gamma a$. Thus, $a \in a\Gamma S\Gamma a \subseteq (a \cup S\Gamma a)\Gamma a = a\Gamma a \cup S\Gamma a\Gamma a$. Hence, S is a left regular Γ-semigroup. □

THEOREM 3.3. *Let S be a right multiplication Γ-semigroup. If S is regular such that every left Γ-ideal is Γ-ideal, then S is semisimple.*

Proof. Let S be a right multiplication Γ-semigroup. For any $a \in S$, we have $a\Gamma S \subseteq S\Gamma a\Gamma S$ by Proposition 2.11. Now, $a\Gamma S = C\Gamma(S\Gamma a\Gamma S)$ for some right Γ-ideal C. Since S is regular, it follows that

$$a\Gamma S = C\Gamma(S\Gamma a\Gamma S) = C\Gamma(S\Gamma a\Gamma S)\Gamma(S\Gamma a\Gamma S) \subseteq a\Gamma S\Gamma S\Gamma a \subseteq a\Gamma S\Gamma a.$$

Since every left Γ-ideal is Γ-ideal, $S\Gamma a\Gamma S \subseteq S\Gamma a$, so that

$$a\Gamma S \subseteq a\Gamma S\Gamma a\Gamma S \subseteq \langle a\rangle \Gamma \langle a\rangle .$$

By Proposition 2.10(ii), $a \in a\Gamma S$ and so $a \in \langle a\rangle \Gamma \langle a\rangle$. Thus, S is a semisimple Γ-semigroup. □

REMARK 3.4. It is observed that there are right multiplication Γ-semigroups that are not regular Γ-semigroups. The semigroup $S = \{1, x, y\}$ with $\Gamma = \{\gamma\}$ is examined in the table below to illustrate this:

γ	1	x	y
1	1	1	1
x	1	y	1
y	y	y	y

By Remark 2.9, it can be easily verified that S is a right multiplication Γ-semigroup. However, S is not regular since x is not a regular element.

Following Satyanarayana's definition of principal right ideal semigroup in [19], we may define:

DEFINITION 3.5. A Γ-semigroup S is called a *principal right ideal Γ-semigroup* if every right ideal R is principal, that is, $R = x \cup x\Gamma S$ for some $x \in S$.

THEOREM 3.6. *A principal right ideal Γ-semigroup S is a right multiplication Γ-semigroup if and only if S contains a left identity.*

Proof. Let S be a right multiplication Γ-semigroup such that $S = x \cup x\Gamma S$ for some $x \in S$. By Proposition 2.10(i), $S = S\Gamma S$ so that $S = e\Gamma S$ for some α-idempotent element $e \in S$ where $e = x$ or $x\alpha s$ for some $s \in S$ and $\alpha \in \Gamma$. Clearly, e acts as a left identity in S.

Conversely, let A and B be any two right Γ-ideals of S with $A \subseteq B$. Then $A = a\Gamma S$ and $B = b\Gamma S$ for some $a, b \in S$, where $a = c\alpha b$ for some $c \in S$ and $\alpha \in \Gamma$. Now, setting $C = c\Gamma S$, we have $A = C\Gamma B$, which shows that S is a right multiplication Γ-semigroup. □

THEOREM 3.7. *A semisimple* Γ*-semigroup* S *is a right multiplication* Γ*-semigroup if and only if* $a \in (a\Gamma S)\Gamma(a\Gamma S)$ *for every* $a \in S$.

Proof. Let S be a right multiplication Γ-semigroup. Then for any $a \in S$, $a\Gamma S \subseteq S\Gamma a\Gamma S$. Hence, $a\Gamma S = B\Gamma(S\Gamma a\Gamma S)$ for some right Γ-ideal B. Since every Γ-ideal is idempotent in a semisimple Γ-semigroup, we have:

$$\begin{aligned} a\Gamma S &= B\Gamma(S\Gamma a\Gamma S) = B\Gamma(S\Gamma a\Gamma S)\Gamma(S\Gamma a\Gamma S) \\ &= a\Gamma S\Gamma(S\Gamma a\Gamma S) \subseteq (a\Gamma S)\Gamma(a\Gamma S). \end{aligned}$$

By Proposition 2.10(ii), $a \in a\Gamma S$ and so $a \in (a\Gamma S)\Gamma(a\Gamma S)$.

Conversely, suppose $a \in (a\Gamma S)\Gamma(a\Gamma S)$ for every $a \in S$. Let

$$\langle a\rangle = a \cup a\Gamma S \cup S\Gamma a \cup S\Gamma a\Gamma S.$$

Then for any $a \in \langle a\rangle$, $a \in a\Gamma S\Gamma a\Gamma S = (a\Gamma S)\Gamma(a\Gamma S) \subseteq \langle a\rangle \Gamma \langle a\rangle \subseteq \langle a\rangle$ that is, $\langle a\rangle = \langle a\rangle \Gamma \langle a\rangle$. Thus, S is semisimple. □

DEFINITION 3.8. Let S_1 and S_2 be Γ-semigroups. The *direct product* of S_1 and S_2 denoted by $S_1 \times S_2$ is defined with respect to the following operation:

$$(a_1, b_1)\alpha(a_2, b_2) = (a_1\alpha a_2, b_1\alpha b_2)$$

for all $a_1, a_2 \in S_1,\ b_1, b_2 \in S_2$ and $\alpha \in \Gamma$.

REMARK 3.9. The direct product of two right multiplication Γ-semigroups need not be a right multiplication Γ-semigroup in general.

The next example verifies Remark 2.9.

EXAMPLE 3.10. Let $\mathbb{Z}_2 = \{0, 1\}$ and $\mathbb{Z}_4 = \{0, 1, 2, 3\}$ be Γ-semigroups with $\Gamma = \{\alpha\}$ as shown in the tables below:

α	0	1
0	0	0
1	1	1

α	0	1	2	3
0	0	0	0	0
1	0	1	1	0
2	0	2	2	0
3	3	3	3	3

Clearly, $\mathbb{Z}_2$ and $\mathbb{Z}_4$ are right multiplication Γ-semigroups while the direct product $\mathbb{Z}_2 \times \mathbb{Z}_4$ is not. Let

$$\begin{aligned} \mathbb{Z}_2 \times \mathbb{Z}_4 &= \{(0,0),(0,1),(0,2),(0,3),(1,0),(1,1),(1,2),(1,3)\}, \\ \mathbb{Z}_2 \times 2\mathbb{Z}_4 &= \{(0,0),(0,2),(1,0),(1,2)\}, \\ \{0\} \times \mathbb{Z}_4 \cup \mathbb{Z}_2 \times 2\mathbb{Z}_4 &= \{(0,0),(0,1),(0,2),(0,3),(1,0),(1,2)\} \end{aligned}$$

and

$$\mathbb{Z}_2 \times \mathbb{Z}_4 \setminus \{2\} = \{(0,0),(0,1),(0,3),(1,0),(1,1),(1,3)\}.$$

Now,

$$\mathbb{Z}_2 \times 2\mathbb{Z}_4\Gamma\mathbb{Z}_2 \times \mathbb{Z}_4 = \{(0,0),(0,2),(1,0),(1,2)\} = \mathbb{Z}_2 \times 2\mathbb{Z}_4,$$

$$\begin{aligned} \{0\}\times\mathbb{Z}_4\cup\mathbb{Z}_2\times 2\mathbb{Z}_4\Gamma\mathbb{Z}_2\times\mathbb{Z}_4 &= \{(0,0),(0,1),(0,2),(0,3),(1,0),(1,2)\} \\ &= \{0\}\times\mathbb{Z}_4\cup\mathbb{Z}_2\times 2\mathbb{Z}_4, \\ \text{and}\quad \mathbb{Z}_2\times\mathbb{Z}_4\setminus\{2\}\Gamma\mathbb{Z}_2\times\mathbb{Z}_4, &= \{(0,0),(0,1),(0,3),(1,0),(1,1),(1,3)\} \\ &= \mathbb{Z}_2\times\mathbb{Z}_4\setminus\{2\}. \end{aligned}$$

This shows that $\mathbb{Z}_2\times 2\mathbb{Z}_4$, $\{0\}\times\mathbb{Z}_4\cup\mathbb{Z}_2\times 2\mathbb{Z}_4$ and $\mathbb{Z}_2\times\mathbb{Z}_4\setminus\{2\}$ are all right Γ-ideals of $\mathbb{Z}_2\times\mathbb{Z}_4$. Since $\mathbb{Z}_2\times 2\mathbb{Z}_4 \neq \mathbb{Z}_2\times\mathbb{Z}_4\setminus\{2\}\Gamma(\{0\}\times\mathbb{Z}_4\cup\mathbb{Z}_2\times 2\mathbb{Z}_4)$, therefore $\mathbb{Z}_2\times\mathbb{Z}_4$ is not a right multiplication Γ-semigroup.

The following theorems give a condition for the direct product of two right multiplication Γ-semigroups to be a right multiplication Γ-semigroup.

Theorem 3.11. *Let S_1 and S_2 be right multiplication Γ-semigroups. If $(a,b)\in((a,b)\Gamma S_1\times S_2)\Gamma((a,b)\Gamma S_1\times S_2)$ for every $(a,b)\in S_1\times S_2$, then $S_1\times S_2$ is a right multiplication Γ-semigroup.*

Proof. The proof is an analogue of [[1], Propositions 4.1]. □

Theorem 3.12. *Let S_1 and S_2 be Γ-semigroups. If $S_1\times S_2$ is regular, then $S_1\times S_2$ is a right multiplication Γ-semigroup.*

Proof. The proof is an analogue of [[1], Propositions 4.2]. □

Remark 3.13. Similar characterisations of the above results presented in this section can be proved for left multiplication Γ-semigroups

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Joseph Sarwuan Tarka University
Department of Mathematics
Makurdi, Benue State, Nigeria
E-mail: remsonjay@yahoo.com
E-mail: awolola.johnson@uam.edu.ng
https://orcid.org/0000-0001-7582-247X

Federal University Lokoja
Department of Mathematics
Kogi State, Nigeria
E-mail: musa.ibrahim@fulokoja.edu.ng
Ahmadu Bello University
Department of Mathematics
Zaria, Kaduna State, Nigeria
E-mail: atimam@abu.edu.ng