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WEAK SOLUTION FOR BIHARMONIC EQUATION WITH NAVIER BOUNDARY CONDITIONS

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Abstract. The aim of this paper is to study the existence and uniqueness of weak solution for the problem

$$(1)\qquad \begin{gathered} Lu(x) - \mu u(x) f(x) = -g\big(x, u(x)\big) \text{ in } \Omega, \\ u(x) = \Delta u(x) = 0 \text{ on } \partial\Omega, \end{gathered}$$

in $W^{2,2}(\Omega, v) \cap W_0^{1,2}(\Omega, v)$ where,

$$Lu(x) = -\sum_{i,j=1}^{n} D_j\big(a_{ij}(x) D_i u(x)\big) + \Delta\big[v(x)\Delta u(x)\big],$$

with $\mu \in \mathbb{R}$, $\Omega \subset \mathbb{R}^n$ is bounded and open. Here, the functions $f : \Omega \to \mathbb{R}$ and $g : \Omega \times \mathbb{R} \to \mathbb{R}$ satisfy the suitable hypotheses.

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1. INTRODUCTION

We prove the existence and uniqueness of weak solution for the problem

$$(2)\qquad \begin{gathered} Lu(x) - \mu u(x) f(x) = -g(x, u(x)) \text{ in } \Omega, \\ u(x) = \Delta u(x) = 0 \text{ on } \partial\Omega, \end{gathered}$$

in $W^{2,2}(\Omega, v) \cap W_0^{1,2}(\Omega, v)$. Here,

$$(3)\qquad Lu(x) = -\sum_{i,j=1}^{n} D_j(a_{ij}(x) D_i u(x)) + \Delta[v(x)\Delta u(x)],$$

with $\mu \in \mathbb{R}$, Ω is an open bounded set in $\mathbb{R}^n$ and the a_{ij} are real-valued, measurable functions. The coefficient matrix (a_{ij}) is symmetric and fulfill the ellipticity condition

$$(4)\qquad \lambda|\ \rho\ |^2 v(x) \leq \sum_{i,j=1}^{n} a_{ij}(x)\rho_i\rho_j \leq \Lambda|\ \rho\ |^2 v(x),$$

for almost every x in Ω, each $\rho \in \mathbb{R}^n$, $\lambda > 0, \Lambda > 0$ and v is a weight function.

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Much has been written in past years about the existence and multiplicity of solutions for nonlinear second order elliptic partial differential equations in bounded and unbounded domains of $\mathbb{R}^n$. Interest in higher order nonlinear problems has recently grown due to exciting and promising developments, particularly for fourth order equations. Semilinear elliptic partial differential equations with biharmonic operator appear in the study of bio-physics, continuum mechanics, differential geometry. In [1], M. Bhakta has studied about the existence and nonexistence of positive solution to the specific semilinear elliptic biharmonic problem with singular potential in space $W^{2,2} \cap W_0^{1,2}(\Omega)$. Applying an additional condition on the equation, the author has established the uniqueness of the positive solution. A similar equation has been studied by M. Pérez-Llanos and A. Primo in [9], where the authors have solved the semilinear biharmonic boundary value problem with the optimal exponent term.

The weight v is a function which is integrable on every compact subset of $\mathbb{R}^n$ and $v > 0$ for almost every x in $\mathbb{R}^n$. Through integration, each weight v yields measure on $\mathbb{R}^n$'s measurable subsets. A_p-weights, which was presented by B. Muckenhoupt [8], is a notable category of weights. These classes offer a wide range of applications in harmonic analysis (we refer [11]).

For parabolic and elliptic partial differential equations, the Sobolev spaces without weight appear as spaces of solutions. It is reasonable to search for solution in weighted Sobolev spaces for the equations having coefficients with different forms of singularities. Many research papers have been published in the recent few years that deal with the $W^{2,p}(\Omega, v) \cap W_0^{1,p}(\Omega, v)$ solution of elliptic partial differential equations with p-biharmonic operator and some specific nonlinearities. We list several of them [3,4,10]. Similar type of problem was studied by A.C. Cavalheiro in [2] involving the Laplacian operator in the space $W_0^{1,2}(\Omega, v)$. We apply the idea of it in studying the biharmonic problem(2). In [3], the author has solved the Navier problem with (p, q)-biharmonic operators to find its existence of weak solution. We refer to [7] for the existence of solution of a class of elliptic problems on the Sierpinski gasket.

In Section 2, we introduce the known results, necessary definitions and hypotheses which we have used subsequently. The main result is proved in the Section 3.

2. PRELIMINARIES

Suppose that v be a locally integrable nonnegative function in $\mathbb{R}^n$ and let $0 < v < \infty$ almost everywhere. We say v is an A_p-weight, if there is a constant B such as

$$\left(\frac{1}{\mid D \mid}\int_D v \mathrm{d}x\right)\left(\frac{1}{\mid D \mid}\int_D v^{1/(1-p)} \mathrm{d}x\right)^{p-1} \leq B,$$

for every ball D contained in $\mathbb{R}^n$, where $\mid . \mid$ stands for Lebesgue measure.

Definition 2.1. We denote $X = W^{2,2}(\Omega, v) \cap W_0^{1,2}(\Omega, v)$ with the norm

$$\| u \|_X = \left(\int_\Omega | \nabla u |^2 v \mathrm{d}x \ + \ \int_\Omega | \Delta u |^2 v \mathrm{d}x \right)^{1/2}.$$

Definition 2.2. If a function $u \in X$ satisfies

$$\int_\Omega a_{ij} D_i u D_j \varphi \mathrm{d}x \ + \int_\Omega \Delta u \Delta \varphi v \mathrm{d}x - \mu \int_\Omega u f \varphi \mathrm{d}x \ = \ - \int_\Omega g \varphi \mathrm{d}x, \tag{5}$$

for every $\varphi \in X$ then, (2) has a weak solution.

Proposition 2.3. *Suppose $v \in A_p$ and $\Omega \subset \mathbb{R}^n$, which is bounded, open. There exist a positive δ and constant C_Ω in such a way that every k fulfilling $1 \leq k \leq \frac{n}{n-1} + \delta$ and for every $u \in C_0^\infty(\Omega)$, we have*

$$C_\Omega \| \nabla u \|_{L^p(\Omega,v)} \geq \| u \|_{L^{kp}(\Omega,v)}.$$

For proof, we refer the Theorem 1.3 of [5].

Proposition 2.4. *Let $v \in A_p$ and $\Omega \subset \mathbb{R}^n$, which is bounded and open. If u_n converges to u in $L^p(\Omega, v)$ then, there is a function $\Phi \in L^p(\Omega, v)$ and a subsequence $\{u_{n_k}\}$ in such a way that:*

(1) *$\Phi(x) \geq | u_{n_k}(x) |$ almost everywhere on Ω;*
(2) *$u_{n_k}(x) \to u(x)$ as $n_k \to \infty$ almost everywhere on Ω.*

For proof, we refer the Theorem 2.8.1 in [6].

Remark 2.5. If $v \in A_2$, then

$$\| u \|_{L^2(\Omega,v)} \leq C_\Omega \| \nabla u \|_{L^2(\Omega,v)} \leq C_\Omega \| u \|_X,$$

so $X \subset L^2(\Omega, v)$ is continuous embedding.

Definition 2.6. Suppose $H : X \to X^*$ be an operator, where X is a real Banach space.

(1) H is angle-bounded iff H is monotone, linear and we have a nonnegative constant β with the aim of

$$| \langle Hu_1, u_2 \rangle_X - \langle Hu_2, u_1 \rangle_X |^2 \leq \beta \langle Hu_1, u_1 \rangle_X \langle Hu_2, u_2 \rangle_X,$$

where u_1, u_2 belong to X and $\langle f, u \rangle_X$ denotes the value of $f(u)$.

(2) H is monotone if and only if

$$\langle Hu_1 - Hu_2, u_1 - u_2 \rangle_X \geq 0,$$

where $u_1, u_2 \in X$.

(3) H is hemicontinuous iff

$$s \mapsto \langle H(u_1 + su_2), u_3 \rangle_X,$$

is continuous on $[0, 1]$, where $u_1, u_2, u_3 \in X$.

Proposition 2.7. *Suppose X is a Banach space, which is real and separable. Suppose operators $F : X^* \to X$ and $K : X \to X^*$ satisfy the assumptions given below:*

(1) *F is monotone and hemicontinuous;*
(2) *K is angle-bounded, monotone and linear.*
Then, $u + KFu = 0$ has only one solution u in X^*.

We refer to [13, Theorem 28 A] for the proof.

For further study, we require the following hypotheses. Let $v \in A_2$.

(H1) Suppose $g : \Omega \times \mathbb{R} \to \mathbb{R}$ satisfies the Carathéodory condition.

(H2) The increasing function $t \mapsto g(x,t)$ on $\mathbb{R}$ for every $x \in \Omega$.

(H3) f_1 and f_2 are two nonnegative functions such as; $f_1 \in L^2(\Omega, v) \cap L^2(\Omega, v^{-1})$, $f_2 \in L^\infty(\Omega)$ and $f_2/v \in L^\infty(\Omega)$ so as

$$| g(x,t) | \leq f_1(x) + f_2(x) \mid t \mid .$$

(H4) $f/v \in L^\infty(\Omega)$.

Let us define $B : X \times X \to \mathbb{R}$, where

$$B(u,\varphi) = \int_\Omega a_{ij} D_i u D_j \varphi \mathrm{d}x \ + \int_\Omega \Delta u \Delta \varphi v \mathrm{d}x \ - \ \mu \int_\Omega u f \varphi \mathrm{d}x.$$

The map u is a weak solution of (2) if $u \in X$ satisfies

$$B(u,\varphi) = -\int_\Omega g\varphi \mathrm{d}x,$$

for all $\varphi \in X$.

3. THE MAIN RESULT

We show the existence as well as uniqueness of solution for equation (2) in this section.

THEOREM 3.1. *Suppose* (H1)–(H4) *hold. Let*

$$0 < \lambda - \mu \parallel f/v \parallel_{L^\infty(\Omega)} C_\Omega < 1, \quad \mu > 0,$$

then (2) *has only one solution $u \in X$.*

Proof. **Step 1.**

Let us define $F : L^2(\Omega, v) \to L^2(\Omega, v)$ as

$$(Fu)(x) = g(x, u(x)).$$

We assert that function F is monotone, bounded and continuous.

Applying (H2), we get that $g = g(x,t)$ is increasing function with respect to t, that is, $g(x,t_1) \leq g(x,t_2)$ holds for all $t_1, t_2 \in \mathbb{R}$ with $t_1 \leq t_2$ and for all $x \in \Omega$. Then $\big(g(x,t_1) - g(x,t_2)\big)(t_1 - t_2) \geq 0$, for all $t_1, t_2 \in \mathbb{R}$, $x \in \Omega$. This implies

$$\begin{aligned} &\langle Fu_1 - Fu_2, u_1 - u_2 \rangle_{L^2(\Omega,v)} \\ &= \int_\Omega \big[g\big(x, u_1(x)\big) - g\big(x, u_2(x)\big)\big]\big(u_1(x) - u_2(x)\big)\mathrm{d}x \geq 0, \end{aligned} \tag{6}$$

for all $u_1, u_2 \in L^2(\Omega, v)$. Thus, $F : L^2(\Omega, v) \to L^2(\Omega, v)$ is monotone.

Next, to show F is bounded, we use (H3) and Proposition 2.3, so that

$$\begin{aligned}
\| Fu \|^2_{L^2(\Omega,v)} &= \int_\Omega | F(u(x)) |^2 v\mathrm{d}x \\
&= \int_\Omega | g(x,u(x)) |^2 v\mathrm{d}x \\
&\le \int_\Omega (f_1(x) + f_2(x) \mid u(x) \mid)^2 v\mathrm{d}x \\
&\le 2\int_\Omega (f_1^2(x) + f_2^2(x) | u(x) |^2) v\mathrm{d}x \\
&\le 2\big(\| f_1 \|^2_{L^2(\Omega,v)} + \| f_2 \|^2_{L^\infty(\Omega)} \| u \|^2_{L^2(\Omega,v)}\big).
\end{aligned} \tag{7}$$

So, F is bounded.

Now, to show the continuity of F, let $u_n \to u$ in $L^2(\Omega, v)$ as $n \to \infty$. We have to verify that $Fu_n \to Fu$ in $L^2(\Omega, v)$. Applying Theorem 2.4, there exist a function $\Phi \in L^2(\Omega, v)$ and a subsequence $\{u_{n_k}\}$ such that $| u_{n_k}(x) | \le \Phi(x)$, almost everywhere in Ω and $u_{n_k}(x) \to u(x)$, almost everywhere in Ω. Thus, we obtain

$$\begin{aligned}
\| Fu_{n_k} - Fu \|^2_{L^2(\Omega,v)} &= \int_\Omega | Fu_{n_k}(x) - Fu(x) |^2 v\mathrm{d}x \\
&= \int_\Omega | g(x,u_{n_k}) - g(x,u) |^2 v\mathrm{d}x \\
&\le 2\int_\Omega \Big(| g(x,u_{n_k}) |^2 + | g(x,u) |^2 \Big) v\mathrm{d}x \\
&\le 2\Big[\int_\Omega \big(f_1(x) + f_2(x) \mid u_{n_k} \mid \big)^2 v\mathrm{d}x \\
&+ \int_\Omega \big(f_1(x) + f_2(x) \mid u \mid \big)^2 v\mathrm{d}x \Big] \\
&\le C\Big(\| f_1 \|^2_{L^2(\Omega,v)} + \| f_2 \|^2_{L^\infty(\Omega)} \| \Phi \|^2_{L^2(\Omega,v)} \Big).
\end{aligned}$$

Applying (H1), we get $Fu_n(x) = g\big(x, u_n(x)\big) \to g\big(x, u(x)\big) = Fu(x)$ when $n \to +\infty$. Hence, using the Dominated Convergence theorem, we get

$$\| Fu_{n_k} - Fu \|_{L^2(\Omega,v)} \to 0,$$

that is, $Fu_{n_k} \to Fu$ in $L^2(\Omega, v)$. Due to the convergence principle in Banach spaces, we can show that $Fu_n \to Fu$ in $L^2(\Omega, v)$. Thus, F is continuous.

Step 2. We claim that the map $B : X \times X \to \mathbb{R}$ is bounded, bilinear and strongly positive.

Since, we have $| a_{ij}(x) | \leq \Lambda v(x)$ for almost every $x \in \Omega$. Using Proposition 2.3 and (H4),

$$
\begin{aligned}
&| B(u,\varphi) | \\
&= \left| \int_\Omega a_{ij} D_i u D_j \varphi \mathrm{d}x + \int_\Omega \Delta u \Delta \varphi v \mathrm{d}x - \mu \int_\Omega u f \varphi \mathrm{d}x \right| \\
&\leq \int_\Omega | a_{ij} || D_i u || D_j \varphi | \mathrm{d}x + \int_\Omega | \Delta u || \Delta \varphi | v \mathrm{d}x \\
&+ | \mu | \int_\Omega | u || f/v || \varphi | v \mathrm{d}x \\
&\leq \int_\Omega \Lambda | D_i u || D_j \varphi | v \mathrm{d}x + \int_\Omega | \Delta u || \Delta \varphi | v \mathrm{d}x \\
&+ | \mu | \| f/v \|_{L^\infty(\Omega)} \int_\Omega | u || \varphi | v \mathrm{d}x \\
&\leq \Lambda \left(\int_\Omega | D_i u |^2 v \mathrm{d}x \right)^{1/2} \left(\int_\Omega | D_j \varphi |^2 v \mathrm{d}x \right)^{1/2} \\
&+ \left(\int_\Omega | \Delta u |^2 v \mathrm{d}x \right)^{1/2} \left(\int_\Omega | \Delta \varphi |^2 v \mathrm{d}x \right)^{1/2} \\
&+ | \mu | \| f/v \|_{L^\infty(\Omega)} \left(\int_\Omega | u |^2 v \mathrm{d}x \right)^{1/2} \left(\int_\Omega | \varphi |^2 v \mathrm{d}x \right)^{1/2} \\
&\leq \Lambda \| u \|_X \| \varphi \|_X + \| u \|_X \| \varphi \|_X + | \mu | \| f/v \|_{L^\infty(\Omega)} C_\Omega^2 \| u \|_X \| \varphi \|_X \\
&= (\Lambda + 1 + | \mu | \| f/v \|_{L^\infty(\Omega)} C_\Omega^2) \| u \|_X \| \varphi \|_X \\
&= C^* \| u \|_X \| \varphi \|_X,
\end{aligned}
$$

where $C^* = \big(\Lambda + 1 + | \mu | \| f/v \|_{L^\infty(\Omega)} C_\Omega^2\big)$. Hence, the bilinear form B is bounded.

We obtain following by applying the inequality (4), (H4), Remark 2.5 and Proposition 2.3,

$$
\begin{aligned}
B(u,u) &= \int_\Omega a_{ij} D_i u D_j u \mathrm{d}x + \int_\Omega (\Delta u)(\Delta u) v \mathrm{d}x - \mu \int_\Omega u f u \mathrm{d}x \\
&= \int_\Omega a_{ij} D_i u D_j u \mathrm{d}x + \int_\Omega | \Delta u |^2 v \mathrm{d}x - \mu \int_\Omega u^2 f/v \, v \mathrm{d}x \\
&\geq \lambda \int_\Omega | \nabla u |^2 v \mathrm{d}x + \int_\Omega | \Delta u |^2 v \mathrm{d}x - \mu \| f/v \|_{L^\infty(\Omega)} \int_\Omega u^2 v \mathrm{d}x \\
&\geq \lambda \int_\Omega | \nabla u |^2 v \mathrm{d}x + \int_\Omega | \Delta u |^2 v \mathrm{d}x - \mu \| f/v \|_{L^\infty(\Omega)} C_\Omega \int_\Omega | \nabla u |^2 v \mathrm{d}x \\
&= \left(\lambda - \mu \| f/v \|_{L^\infty(\Omega)} C_\Omega \right) \int_\Omega | \nabla u |^2 v \mathrm{d}x + \int_\Omega | \Delta u |^2 v \mathrm{d}x
\end{aligned}
$$

$$= \left(\lambda - \mu \parallel f/v \parallel_{L^\infty(\Omega)} C_\Omega\right) \int_\Omega |\nabla u|^2 v\mathrm{d}x + \parallel u \parallel_X^2 - \int_\Omega |\nabla u|^2 v\mathrm{d}x$$
$$= \left(\lambda - \mu \parallel f/v \parallel_{L^\infty(\Omega)} C_\Omega - 1\right) \int_\Omega |\nabla u|^2 v\mathrm{d}x + \parallel u \parallel_X^2$$
$$= -\left(-\lambda + \mu \parallel f/v \parallel_{L^\infty(\Omega)} C_\Omega + 1\right) \int_\Omega |\nabla u|^2 v\mathrm{d}x + \parallel u \parallel_X^2$$
$$\geq -\left(-\lambda + \mu \parallel f/v \parallel_{L^\infty(\Omega)} C_\Omega + 1\right) \parallel u \parallel_X^2 + \parallel u \parallel_X^2$$
$$= \left(\lambda - \mu \parallel f/v \parallel_{L^\infty(\Omega)} C_\Omega\right) \parallel u \parallel_X^2 = C_1 \parallel u \parallel_X^2,$$

where,

$$C_1 = \left(\lambda - \mu \parallel f/v \parallel_{L^\infty(\Omega)} C_\Omega\right) > 0 \text{ and } \left(-\lambda + \mu \parallel f/v \parallel_{L^\infty(\Omega)} C_\Omega + 1\right) > 0.$$

Therefore, the bilinear map B is strongly positive, if

$$0 < \lambda - \mu \parallel f/v \parallel_{L^\infty(\Omega)} C_\Omega < 1.$$

Step 3. We study the following linear problem

$$\begin{aligned} Lu(x) - \mu u(x) f(x) &= h(x) \text{ in } \Omega, \\ u(x) = \Delta u(x) &= 0 \text{ on } \partial\Omega. \end{aligned} \tag{8}$$

Here, $h(x) \equiv -g(x, u(x))$. Applying Theorem 22.C in [12], we obtain a unique solution $u \in X \subset L^2(\Omega, v)$ for

$$B(u, \varphi) = -\int_\Omega h(x)\varphi(x)\mathrm{d}x,$$

where, $\varphi \in X$. By putting $u = Kh$ and using Corollary 22.20 from [12], $K : L^2(\Omega, v) \to L^2(\Omega, v)$ is angle-bounded, compact, monotone and linear. Hence, the problem (2) is equivalent to the operator equation

$$u + KFu = 0, \text{ for all } u \in L^2(\Omega, v). \tag{9}$$

Now, applying Proposition 2.7, we get that Equation (9) has a unique solution. Thus, we can conclude that the solution to the problem (2) is unique. □

Remark 3.2. In particular, for $\varphi = u \in X$, we obtain

$$B(u, u) = -\int_\Omega g(x, u(x))u(x)\mathrm{d}x.$$

From Step 2, we get

$$B(u, u) \geq C_1 \parallel u \parallel_X^2.$$

Applying (H3) and Remark 2.5, we have

$$\begin{aligned}
\left| \int_\Omega g(x,u)u\mathrm{d}x \right| &\leq \int_\Omega \mid g(x,u) \mid\mid u \mid \mathrm{d}x \\
&\leq \int_\Omega \left(f_1 + f_2 \mid u \mid \right) \mid u \mid \mathrm{d}x \\
&= \int_\Omega f_1 v^{-1/2} \mid u \mid v^{1/2}\mathrm{d}x + \int_\Omega f_2 v^{-1} \mid u \mid^2 v\mathrm{d}x \\
&\leq \left(\int_\Omega {f_1}^2 v^{-1}\mathrm{d}x \right)^{1/2} \left(\int_\Omega \mid u \mid^2 v\mathrm{d}x \right)^{1/2} + \parallel f_2/v \parallel_{L^\infty(\Omega)} \int_\Omega \mid u \mid^2 v\mathrm{d}x \\
&\leq \parallel f_1 \parallel_{L^2(\Omega,v^{-1})} \parallel u \parallel_{L^2(\Omega,v)} + \parallel f_2/v \parallel_{L^\infty(\Omega)} \parallel u \parallel^2_{L^2(\Omega,v)} \\
&\leq C_\Omega \parallel f_1 \parallel_{L^2(\Omega,v^{-1})} \parallel u \parallel_X + C_\Omega^2 \parallel f_2/v \parallel_{L^\infty(\Omega)} \parallel u \parallel_X^2 .
\end{aligned}$$

Hence,

$$C_1 \parallel u \parallel_X^2 \leq C_\Omega \parallel f_1 \parallel_{L^2(\Omega,v^{-1})} \parallel u \parallel_X + C_\Omega^2 \parallel f_2/v \parallel_{L^\infty(\Omega)} \parallel u \parallel_X^2 .$$

Thus,

$$\parallel u \parallel_X \leq \frac{C_\Omega \parallel f_1 \parallel_{L^2(\Omega,v^{-1})}}{C_1 - C_\Omega^2 \parallel f_2/v \parallel_{L^\infty(\Omega)}} .$$

Now, putting $\frac{C_\Omega}{C_1 - C_\Omega^2 \| f_2/v \|_{L^\infty(\Omega)}} = C^{**}$, we have

$$\parallel u \parallel_X \leq C^{**} \parallel f_1 \parallel_{L^2(\Omega,v^{-1})} .$$

$C^{**} > 0$ if $C_1 - C_\Omega^2 \parallel f_2/v \parallel_{L^\infty(\Omega)} > 0$.

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