GROUP GRADED ENDOMORPHISM ALGEBRAS AND MORITA EQUIVALENCES

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Abstract. We prove a group graded Morita equivalences version of the “butterfly theorem” on character triples. This gives a method to construct an equivalence between block extensions from another related equivalence.

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1. INTRODUCTION AND PRELIMINARIES

The Butterfly theorem, as stated by B. Späth in [3, Theorem 2.16], gives the possibility to construct certain relations between character triples. The result is very useful in obtaining reduction methods for the local-global conjectures in modular representation theory of finite groups. In this paper, we consider group graded Morita equivalences between block extensions and we obtain an analogue of [3, Theorem 2.16]. Our main result, Theorem 4.2, shows how to construct a group graded Morita equivalence from a given one, under very similar assumptions to those in [3].

In general, our notations and assumptions are standard and follow [2]. To introduce our context, let $G$ be a finite group, $N$ a normal subgroup of $G$, and denote by $\bar{G}$ the factor group $G/N$. Let $A = \bigoplus_{\bar{g} \in \bar{G}} A_{\bar{g}}$ be a strongly $\bar{G}$-graded $O$-algebra with the identity component $B := A_1$, where $(k, O, k)$ is a $p$-modular system. For a subgroup $\bar{H}$ of $\bar{G}$, we denote by $A_{\bar{H}} := \bigoplus_{\bar{g} \in \bar{H}} A_{\bar{g}}$ the truncation of $A$ from $\bar{G}$ to $\bar{H}$.

For the sake of simplicity, in this article we will mostly consider only crossed products, also because the generalization of the statements to the case of strongly graded algebras is a mere technicality. Recall that, if $A$ is a crossed product, we can chose an invertible homogeneous element $u_{\bar{g}}$ in the component $A_{\bar{g}}$, for all $\bar{g} \in \bar{G}$.

Our main example for a $\bar{G}$-graded crossed product is obtained as follows: Regard $OG$ as a $\bar{G}$-graded algebra with the 1-component $O N$. Let $b \in Z(ON)$ be a $\bar{G}$-invariant block idempotent. We denote $A := bOG$ and $B := bON$. Then the block extension $A$ is a $\bar{G}$-graded crossed product, with 1-component $B$.

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The paper is organized as follows. In Section 2, we recall from [2] the main facts on group graded Morita equivalences and we state a graded variant of the second Morita Theorem [1, Theorem 12.12]. In Section 3, we show that there is a natural map, compatible with Morita equivalences, from the centralizer $C_A(B)$ of $B$ in $A$ to the endomorphism algebra of a $\mathcal{G}$-graded $A$-module induced from a $B$-module. In the last section, we prove that a Morita equivalence between the 1-components of two block extensions always lifts to a graded equivalence between certain centralizer algebras. This is the main ingredient in the proof of our main result, Theorem 4.2.

2. GROUP GRADED MORITA EQUIVALENCES

Let $A = \bigoplus_{g \in \mathcal{G}} A_g$ and $A' = \bigoplus_{g \in \mathcal{G}} A'_g$ be strongly $\mathcal{G}$-graded algebras, with the 1-components $B$ and $B'$ respectively.

It is clear that $A \otimes \mathcal{O} A'^{\text{op}}$ is a $\mathcal{G} \times \mathcal{G}$-graded algebra. Let

$$\delta(\mathcal{G}) := \{(\bar{g}, \bar{g}) \mid \bar{g} \in \mathcal{G}\}$$

be the diagonal subgroup of $\mathcal{G} \times \mathcal{G}$, and let $\Delta$ be the diagonal subalgebra of $A \otimes \mathcal{O} A'^{\text{op}}$

$$\Delta := (A \otimes \mathcal{O} A'^{\text{op}})_{\delta(\mathcal{G})} = \bigoplus_{\bar{g} \in \mathcal{G}} A_{\bar{g}} \otimes A'_{\bar{g}}.$$

Then $\Delta$ is a $\mathcal{G}$-graded algebra, with 1-component $\Delta_1 = B \otimes \mathcal{O} B'^{\text{op}}$.

Let $M$ be a $(B, B')$-bimodule, or, equivalently, $M$ is a $B \otimes \mathcal{O} B'^{\text{op}}$-module, thus a $\Delta_1$-module. Let $M^* := \text{Hom}_B(M, B)$ be its $B$-dual. Note that if $B$ is a symmetric algebra, then we have the isomorphism

$$M^* := \text{Hom}_B(M, B) \simeq \text{Hom}_\mathcal{O}(M, \mathcal{O}),$$

where $\text{Hom}_\mathcal{O}(M, \mathcal{O})$, is the $\mathcal{O}$-dual of $M$.

DEFINITION 2.1. We say that the $\mathcal{G}$-graded $(A, A')$-bimodule $\tilde{M}$ induces a $\mathcal{G}$-graded Morita equivalence between $A$ and $A'$, if $\tilde{M} \otimes_A M^* \simeq A$ as $\mathcal{G}$-graded $(A, A)$-bimodules and that $\tilde{M}^* \otimes_A \tilde{M} \simeq A'$ as $\mathcal{G}$-graded $(A', A')$-bimodules, where the $A$-dual $\tilde{M}^* = \text{Hom}_A(\tilde{M}, A)$ of $\tilde{M}$ is a $\mathcal{G}$-graded $(A', A)$-bimodule.

By [2, Theorem 5.1.2], the following statements are equivalent:

(1) between $B$ and $B'$ we have a Morita equivalence given by the $\Delta_1$-module $M$ and $M$ extends to a $\Delta$-module;

(2) $\tilde{M} := A \otimes_B M$ is a $\mathcal{G}$-graded $(A, A')$-bimodule and $\tilde{M}^* := A' \otimes_B M^*$ is a $\mathcal{G}$-graded $(A', A)$-bimodule, which induce a $\mathcal{G}$-graded Morita equivalence between $A$ and $A'$, given by the functors:

$$A \xleftarrow[\tilde{M}_A \otimes_{A'} \cdot] A' \xrightarrow[\cdot \tilde{M}^*_{A'} \otimes_A \cdot] A'.$$
In this case, by [2, Lemma 1.6.3], we have the natural isomorphisms of \( \tilde{G} \)-graded bimodules
\[
\tilde{M} := A \otimes_B M \simeq M \otimes_B' A' \simeq ((A \otimes A')^{\text{op}}) \otimes_{\Delta} M.
\]

Assume that \( B \) and \( B' \) are Morita equivalent. Then, by the second Morita Theorem [1, Theorem 12.12], we can choose the bimodule isomorphisms
\[
\varphi : M^* \otimes_B M \to B', \quad \psi : M \otimes_B' M^* \to B.
\]
such that
\[
\psi(m \otimes m^*)n = m \varphi(m^* \otimes n), \quad \forall m, n \in M, \ m^* \in M^*
\]
and that
\[
\varphi(m^* \otimes m)n^* = m^* \psi(m \otimes n^*), \quad \forall m^*, n^* \in M^*, \ m \in M.
\]

By the surjectivity of these functions, we may choose finite sets \( I \) and \( J \) and the elements \( m^*_i, n^*_i \in M^* \) and \( m_j, n_i \in M \), for all \( i \in I, \ j \in J \) such that:
\[
\varphi\left(\sum_{j \in J} m^*_j \otimes_B m_j\right) = 1_{B'}, \quad \psi\left(\sum_{i \in I} n_i \otimes_B n^*_i\right) = 1_B.
\]

Assume that \( \tilde{M} \) and \( \tilde{M}^* \) give a \( \tilde{G} \)-graded Morita equivalence between \( A \) and \( A' \). As above, by [1, Theorem 12.12], we can choose the isomorphisms
\[
\tilde{\varphi} : \tilde{M}^* \otimes_A \tilde{M} \to A', \quad \tilde{\psi} : \tilde{M} \otimes_{A'} \tilde{M}^* \to A
\]
of \( \tilde{G} \)-graded bimodules such that
\[
\tilde{\psi}(\tilde{m} \otimes \tilde{m}^*)\tilde{n} = \tilde{m}\tilde{\varphi}(\tilde{m}^* \otimes \tilde{n}), \quad \forall \tilde{m}, \tilde{n} \in \tilde{M}, \ \tilde{m}^* \in \tilde{M}^*
\]
and that
\[
\tilde{\varphi}(\tilde{m}^* \otimes \tilde{m})\tilde{n}^* = \tilde{m}^*\tilde{\psi}(\tilde{m} \otimes \tilde{n}^*), \quad \forall \tilde{m}^*, \tilde{n}^* \in \tilde{M}^*, \ \tilde{m} \in \tilde{M}.
\]
Actually, \( \tilde{\varphi}_1 \) and \( \tilde{\psi}_1 \) are the same with \( \varphi \) and \( \psi \) from before and are \( \Delta \)-linear isomorphisms. Moreover, we have that \( 1_A = 1_B \in B \) and \( 1_{A'} = 1_{B'} \in B' \). Henceforth, we may choose the same finite sets \( I \) and \( J \) and the same elements \( m^*_j, n^*_i \in M^* \) and \( m_j, n_i \in M \), \( \forall i \in I, j \in J \) such that:
\[
\tilde{\varphi}\left(\sum_{j \in J} m^*_j \otimes_B m_j\right) = 1_{B'}, \quad \tilde{\psi}\left(\sum_{i \in I} n_i \otimes_B n^*_i\right) = 1_B.
\]

3. CENTRALIZERS AND GRADED ENDOMORPHISM ALGEBRAS

We will assume that \( A \) and \( A' \) are \( \tilde{G} \)-graded crossed products, although the results of this section can be generalized to strongly graded algebras. Let \( U \in B\text{-mod} \) and \( U' \in B'\text{-mod} \) such that \( U' = M^* \otimes_B U \). We denote
\[
E(U) := \text{End}(A \otimes_B U)^{\text{op}}, \quad E(U') := \text{End}(A' \otimes_{B'} U')^{\text{op}},
\]
the \( \tilde{G} \)-graded endomorphism algebras of the modules induced from \( U \) and \( U' \).
We will prove that there exists a natural $G$-graded algebra homomorphism between the centralizer of $B$ in $A$ and $E(U)$, compatible with $G$-graded Morita equivalences.

**Lemma 3.1.** The map

$$\theta : C_A(B) \rightarrow E(U), \quad \theta(c)(a \otimes u) = ac \otimes u,$$

where $c \in C_A(B)$, $a \in A$ and $u \in U$ is a homomorphism of $G$-graded algebras.

**Proof.** We first need to show that the map is well-defined. For $c \in C_A(B)$, $a \in A$, $b \in B$ and $u \in U$, we have:

$$\theta(c)(ab \otimes Bu) = ab \cdot c \otimes Bu = acb \otimes Bu = \theta(c)(a \otimes Bu).$$

To show that $\theta(c)$ is $A$-linear, let $a' \in A$; we have:

$$\theta(c)(a'a \otimes Bu) = a'ac \otimes Bu = a'(ac \otimes Bu) = a'\theta(c)(a \otimes Bu).$$

To prove that the map is a ring homomorphism, let $c, c' \in C_A(B)$; we have:

$$\theta(c') \theta(c)(a \otimes Bu) = (\theta(c') \circ \theta(c))(a \otimes Bu) = \theta(c') \theta(c)(a \otimes Bu) = \theta(c')ac \otimes Bu = \theta(c')(ac \otimes Bu) = \theta(c)(cc') \theta(c)(a \otimes Bu).$$

Finally, we check that $\theta$ is grade-preserving. Let $a_{\bar{g}} \otimes Bu \in A_{\bar{g}} \otimes BU$ and $c \in C_A(B)_{\bar{h}}$, where $\bar{g}, \bar{h} \in \hat{G}$. Then the definition of $\theta$ says that

$$\theta(c)(a_{\bar{g}} \otimes Bu) = a_{\bar{g}} \cdot c \otimes Bu \in A_{\bar{g} \bar{h}} \otimes BU.$$  

If follows that $\theta(c)$ belongs to $E(U)_{\bar{h}}$. The other properties are obvious. \qed

By [2, Lemma 1.6.3], we have

$$A \otimes_{B'} A' \simeq M \otimes_{B'} A',$$

and we will need an explicit isomorphism between the two. We will choose invertible elements $u_{\bar{g}} \in U(A) \cap A_{\bar{g}}$ and $u'_{\bar{g}} \in U(A) \cap A'_{\bar{g}}$ of degree $\bar{g} \in \hat{G}$. We have that an arbitrary element $a'_{\bar{g}} \in A'_{\bar{g}}$ can be written uniquely in the form $a'_{\bar{g}} = u'_{\bar{g}} b'$, where $b' \in B'$. The desired $\hat{G}$-graded bimodule isomorphism is:

$$\varepsilon : M \otimes_{B'} A' \rightarrow A \otimes_{B'} M \quad m \otimes_{B'} a'_{\bar{g}} \mapsto u_{\bar{g}} \otimes_{B'} B u_{\bar{g}}^{-1} m a'_{\bar{g}}$$

for $m \in M$. We will also need the explicit isomorphism of $G$-graded bimodules

$$\beta : A' \otimes_{B'} M^* \rightarrow M^* \otimes_{B'} A \quad a'_{\bar{g}} \otimes_{B'} m^* \mapsto a'_{\bar{g}} m^* u_{\bar{g}}^{-1} \otimes_{B} u_{\bar{g}}$$

for $m^* \in M^*$. Henceforth we consider the isomorphism of $\hat{G}$-graded $A'$-modules

$$\beta \otimes_{B} id_U : A' \otimes_{B'} M^* \otimes_{B} U \rightarrow M^* \otimes_{B} A \otimes_{B} U.$$
We consider arbitrary elements $a$ for all $\beta$ by the isomorphism $G$

Thus the statement is proved. \hfill $\square$

To prove that the diagram is commutative, let $c \in C_A(B)$, $\phi, \phi' \in C_A(B')$, $u, u' \in U$ and $f \in E(U)$.

Proof. According to Lemma 3.1, we have that $\theta, \theta'$ are homomorphisms of $G$-graded algebras. Moreover, $\phi_1$ and $\phi_2$ are the algebra isomorphisms induced by the $G$-graded Morita equivalence.

To prove that the diagram is commutative, let $c \in C_A(B)_h$, where $h \in G$.

We consider arbitrary elements $a'_g \in A'_g$, where $g \in G$ and $u' = m^* \otimes_B u \in U' = M^* \otimes_B U$. By the above remarks, for all $f \in E(U)$, we have

$$\phi_1(f)(a'_g \otimes_B m^* \otimes_B u) = a'_g m^* u_g^{-1} \otimes_B f(u_g \otimes_B u),$$

hence, for $f = \theta(c) \in E(U)$ we get

$$\phi_1(\theta(c))(a'_g \otimes_B m^* \otimes_B u) = a'_g m^* u_g^{-1} \otimes_B u_g c \otimes_B u.$$

On the other hand, $c' := \phi_2(c) \in C_A'(B')_h$, hence, via the identification given by the isomorphism $\beta$, we have

$$\theta'(\phi_2(c))(a'_g \otimes_B m^* \otimes_B u) = a'_g \phi_2(c \otimes_B m^*) u_g^{-1} \otimes_B u_g c \otimes_B u$$

$$= a'_g \sum_j m^*_j c \otimes_B m_j m^*_h u_h^{-1} u_g^{-1} \otimes_B u_g c \otimes_B u$$

$$= a'_g \sum_j m^*_j \psi(m_j \otimes_B m^*) u_h^{-1} u_g^{-1} \otimes_B u_g c \otimes_B u$$

$$= a'_g \phi_2(c \otimes_B m^*_j m_j m^*_h u_h^{-1} u_g^{-1} \otimes_B u_g c \otimes_B u)$$

Thus the statement is proved. \hfill $\square$
4. THE BUTTERFLY THEOREM FOR $\tilde{G}$-GRADED MORITA EQUIVALENCES

Let $N$ be a normal subgroup of $G$, $G'$ a subgroup of $G$, and $N'$ a normal subgroup of $G'$. We assume that $N' = G' \cap N$ and $G = G'N$, hence $\tilde{G} := G/N \cong G'/N'$. Let $b \in Z(\mathcal{O}N)$ and $b' \in Z(\mathcal{O}N')$ be $\tilde{G}$-invariant block idempotents. We denote

$$A := bOG, \quad A' := b'OOG', \quad B := bON, \quad B' := b'ON'.$$

Then $A$ and $A'$ are strongly $\tilde{G}$-graded algebras, with 1-components $B$ and $B'$ respectively. Additionally, assume that $C_G(N) \subseteq G'$, and denote $\tilde{C}_G(N) := NC_G(N)/N$.

We consider the algebras

$$A := bOG \quad A' := b'OOG', \quad B := bON \quad B' := b'ON'.$$

If $M$ induces a Morita equivalence between $B$ and $B'$, the question that arises is what can we deduce without the additional hypothesis that $M$ extends to a $\Delta$-module. One answer is given by the following proposition.

**Proposition 4.1.** Assume that:

1. $C_G(N) \subseteq G'$.
2. $M$ induces a Morita equivalence between $B$ and $B'$.
3. $zm = mz$ for all $m \in M$ and $z \in Z(N)$.

Then there is a $\tilde{C}_G(N)$-graded Morita equivalence between $C$ and $C'$, induced by the $\tilde{C}_G(N)$-graded $(C,C')$-bimodule

$$C \otimes_B M \simeq M \otimes_B C' \simeq (C \otimes C^{\text{op}}) \otimes_{\Delta(C \otimes C^{\text{op}})} M.$$

**Proof.** Firstly, it is easy to see that our assumption implies that $NC_G(N)/N$ is isomorphic to $N'C_G(N)/N'$. Thus both $C$ and $C'$ are indeed strongly $\tilde{C}_G(N)$-graded algebras.

Now, we prove that there is a $\tilde{C}_G(N)$-graded Morita equivalence between $C$ and $C'$. It suffices to prove that $C \otimes_B M$ is actually a $\tilde{C}_G(N)$-graded $(C,C')$-bimodule.

In view of Lemma 3.1, there exists a $\tilde{G}$-graded algebra homomorphism between $C_A(B)$ and $\text{End}_A(A \otimes_B M)^{\text{op}}$. Moreover, note that $A \otimes_B M$ is a $\tilde{G}$-graded $(A,\text{End}_A(A \otimes_B M)^{\text{op}})$-bimodule, hence by restricting the scalars we obtain that $A \otimes_B M$ is a $\tilde{G}$-graded $(A,C_A(B))$-bimodule. We truncate to
the subgroup $\tilde{C}_G(N)$ of $G$, and we obtain that $A_{G}(N)\otimes B M$ is a $\tilde{C}_G(N)$-
graded $(A_{\tilde{C}}(N), C_A(B)C_{\tilde{C}}(N))$-bimodule, but $A_{\tilde{C}}(N) = b\text{ON}C_G(N) = C$,
hence $M := C\otimes B M$ is a $\tilde{C}_G(N)$-graded $(C, C_A(B)\tilde{C}_G(N))$-bimodule.

We have that $OC_G(N)$ is $\tilde{C}_G(N)$-graded with the 1-component $OZ(N)$ and there is an algebra homomorphism from
$OC_G(N)$ to $C_A(B)$, whose image is evidently included in $C_A(B)\tilde{C}_G(N)$.
Hence, by restricting the scalars, we obtain that $\hat{M}$ is a $\tilde{C}_G(N)$-graded $(C, OC_G(N))$-bimodule. Finally, since $M$
is $(B, B')$-bimodule, where $B' = b'ON'$, we may define on $\hat{M}$ a structure of a
$\tilde{C}_G(N)$-graded $(C, b'ON'C_G(N))$-bimodule, as follows. Let $c \in C$, $m \in M$, $c' \in C_G(N) \subseteq C'$ and $n \in N$
and define $(c \otimes m)c'n = cc' \otimes mn$. To see that this is well-defined, let $z \in Z(N)$, so $c'n = (c'z)(z^{-1}n)$. Then, by assumption
(3), we have

$$(c \otimes m)(c'z)(z^{-1}n) = cc'z \otimes mz^{-1}n = cc'z \otimes mz^{-1}n = cc' \otimes mn.$$ 

Consequently, $\hat{M}$ is a $\tilde{C}_G(N)$-graded $(C, C')$-bimodule. \hfill \Box

Our main result is a version for Morita equivalences of the so-called “butterfly theorem” [3, Theorem 2.16].

**Theorem 4.2.** Let $G$ be another group with normal subgroup $N$ such that
the block $b$ is also $G$-invariant. Assume that:

1. $C_G(N) \subseteq G'$;
2. $\tilde{M}$ induces a $G$-graded Morita equivalence between $A$ and $A'$;
3. $zm = mz$ for all $m \in M$ and $z \in Z(N)$;
4. the conjugation maps $\varepsilon : G \to Aut(N)$ and $\hat{\varepsilon} : G \to Aut(N)$ satisfy
$\varepsilon(G) = \hat{\varepsilon}(G)$.

Denote $\tilde{G}' = \hat{\varepsilon}^{-1}(\varepsilon(G'))$. Then there is a $\tilde{G}/N$-graded Morita equivalence
between $A := b\text{OG}$ and $A' := b'\text{OG}'$.

**Proof.** Consider the following diagram:

\[
\begin{array}{cccc}
\hat{A} := b\text{OG} & A := b\text{OG} & A' := b'\text{OG}' & \hat{A}' := b'\text{OG}' \\
\downarrow \tilde{M} & \downarrow & \downarrow & \downarrow \\
b\text{ON}C_G(N) & \equiv & b'\text{ON}'C_G(N) & \equiv \\
B := \text{ON}b & \equiv & B' := \text{ON}'b'.
\end{array}
\]

By the proof of [3, Theorem 2.16], we have that $C_G(N) \leq \tilde{G}'$, $\tilde{G} = NC_G(N)$ and $N' = N \cap \tilde{G}'$. Note that $NC_G(N)$ is the kernel of the map $G \to Out(N)$
induced by conjugation. Hence the hypothesis $\varepsilon(G) = \hat{\varepsilon}(G)$ implies that $G/NC_G(N) \simeq \tilde{G}/NC_{\tilde{G}}(N)$. It follows that $\tilde{G}/C_{\tilde{G}}(N) \simeq \tilde{G}/C_{\tilde{G}}(N)$. 

Let $C$ and $C'$ be as in Proposition 4.1 and denote $\hat{C} = b\mathcal{O}NC\hat{G}(N)$ and $\hat{C}' = b'\mathcal{O}N'C\hat{G}'(N)$. By Proposition 4.1, we know that the Morita equivalence between $B$ and $B'$ induced by $M$ extends to a $\hat{C}\hat{G}(N)$-graded Morita equivalence between $\hat{C}$ and $\hat{C}'$, induced by $\hat{C}\otimes_B M$.

Let $\mathcal{T} \subset G'$ be a complete set of representatives for the cosets of $NC\hat{G}(N)$ in $G'$. Because $G = NG'$, $\mathcal{T}$ is a complete set of representatives for the cosets of $NC\hat{G}(N)$ in $\hat{G}$.

For any $t \in \mathcal{T}$, we choose $\hat{t} \in \hat{G}'$ such that $\varepsilon(t) = \varepsilon(\hat{t})$. Thus, we obtain a complete set $\hat{T}$ of representatives of $NC\hat{G}(N)$ in $\hat{G}$, so $\hat{T}$ is also a complete set of representatives for the cosets of $NC\hat{G}(N)$ in $\hat{G}$.

We need to define $\hat{\Delta} := \Delta(\hat{A} \otimes \hat{A}^{\text{top}})$-module structure on $M$, knowing that $M$ is $\Delta(A \otimes A^{\text{op}})$-module and a $\Delta(\hat{A} \hat{G}(N) \otimes \hat{A}^{\text{top}}\hat{G}(N))$-module, where

$$\Delta(\hat{A}\hat{G}(N) \otimes \hat{A}^{\text{top}}\hat{G}(N)) \simeq \Delta(A \otimes A^{\text{top}})\hat{G}(N).$$

We define $(\hat{t} \otimes \hat{t}') \cdot m = (t \otimes t') \cdot m$. It is a routine to verify that this definition does not depend on the choices we made and that it gives the required $\hat{\Delta}$-module structure on $M$.

Alternatively, one may argue as follows: The cohomology class $[\hat{\alpha}]$ from $H^2(\hat{G}/N, Z(B)^\times)$ associated to the $\hat{\Delta}_1$-module $M$ satisfies $\text{Res}^{\hat{G}/N}_{NC\hat{G}(N)}[\hat{\alpha}] = 1$, because $M$ extends to a $\hat{\Delta}\hat{G}(N)$-module. It follows that $[\hat{\alpha}] \in \text{ImInf}^{\hat{G}/N}_{NC\hat{G}(N)}$. On the other hand, the class $[\alpha] \in H^2(\hat{G}, Z(B)^\times)$ associated to the $\Delta_1$-module $M$ is trivial, since $M$ extends to a $\Delta$-module. It is easy to see that $(t \otimes t') \otimes M \simeq (t \otimes t') \otimes M$ as $(B,B)$-bimodules, and, since $G/NC\hat{G}(N) \simeq \hat{G}/NC\hat{G}(N)$, we deduce that $[\hat{\alpha}]$ is also trivial, hence $M$ extends to a $\hat{\Delta}$-module. □

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