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ON A GENERALIZED CAYLEY GRAPH OF COLUMN MATRICES OF ELEMENTS OF A FINITE GROUP

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Abstract. Let G be a finite group and let X_m be an $m \times 1$ column matrix of elements of G. Let S be a nonempty subset of G such that $e \notin S$ and $S^{-1} \subseteq S$. If $\mathrm{Cay}(G, S)$ is the usual Cayley graph, whose vertices are all elements of G and two vertices x and y are adjacent if and only if $xy^{-1} \in S$, then the generalized Cayley graph $\mathrm{Cay}_m(G, S)$ is a graph with vertex set consisting of all column matrices X_m, and two vertices X_m and Y_m are adjacent if and only if $X_m[(Y_m)^{-1}]^t \in M(S)$, where ${Y_m}^{-1}$ is a column matrix that each entry is the inverse of the corresponding entry of Y_m, $M(S)$ is an $m \times m$ matrix with all entries in S, $[Y^{-1}]^t$ is the transpose of Y^{-1}, and $m \geq 1$. It is obvious that if $m = 1$, then $\mathrm{Cay}_m(G, S)$ and $\mathrm{Cay}(G, S)$ coincide. In this article, we establish some basic properties of the new graph and determine the structure of $\mathrm{Cay}_m(G, S)$ when $\mathrm{Cay}(G, S)$ is a cycle, C_n, for every $n \geq 3$.

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Key words. Cayley graph, cycle graph, generalized Cayley graph, column matrix.

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