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NEW QUANTUM INEQUALITIES OF HERMITE-HADAMARD TYPE VIA GREEN FUNCTION

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Abstract. In this study, the Hermite-Hadamard inequality for q^{κ_2}-integrals is demonstrated by a new method called the Green Function Technique. For this purpose, we first obtain certain identities. Then, by using these identities, we establish many new inequalities for functions whose second derivative is convex, monotone and concave in absolute value.

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Key words. Hermite-Hadamard inequality, q-integral, quantum calculus, convex function.

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