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CLOSED CO-HOPFIAN MODULES

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Abstract. In this paper, we properly generalize the notion of co-Hopficity for modules to the concept of closed co-Hopficity. A module M is said to be closed co-Hopfian if any injective endomorphism of M has a closed submodule image. The aim of this paper is to study and investigate this class of modules. In addition, some relations for this class with other types of modules are provided.

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Key words. Closed submodule, closed co-Hopfian module, weakly co-Hopfian module.

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