

POSITIVE SOLUTIONS FOR A  $(p, 2)$ -LAPLACIAN STEKLOV  
PROBLEM

ABDELMAJID BOUKHSAS, ABDELLAH ZEROUALI, OMAR CHAKRONE,  
and BELHADJ KARIM

**Abstract.** In this work, we study positive solutions of a Steklov problem driven by the  $(p, 2)$ -Laplacian operator by using the variational method. A sufficient condition for the existence of positive solutions is characterized by the eigenvalues of a linear eigenvalue problem and another nonlinear eigenvalue problem.

**MSC 2010.** 35J20, 35J62, 35J70, 35P05, 35P30.

**Key words.**  $(p, 2)$ -Laplacian, nonlinear boundary conditions.

REFERENCES

- [1] R. Aris, *Mathematical modelling techniques*, Pitman, London, 1978.
- [2] V. Benci, P. d'Avenia, D. Fortunato and L. Pisani, *Solutions in several space dimensions: Derrick's problem and infinitely many solutions*, Arch. Ration. Mech. Anal., **154** (2000), 297–324.
- [3] A. Boukhsas, A. Zerouali, O. Chakrone and B. Karim, *Multiple solutions for a  $(p, q)$ -Laplacian Steklov problem*, An. Univ. Craiova Ser. Mat. Inform., **47** (2020), 357–368.
- [4] A. Boukhsas, A. Zerouali, O. Chakrone and B. Karim, *On a positive solutions for  $(p, q)$ -Laplacian Steklov problem with two parameters*, Bol. Soc. Parana. Mat. (3), **40** (2022), 1–19.
- [5] A. Boukhsas, A. Zerouali, O. Chakrone and B. Karim, *Steklov eigenvalue problems with indefinite weight for the  $(p, q)$ -Laplacian*, to appear.
- [6] L. Cherfilis and Y. Ilyasov, *On the stationary solutions of generalized reaction diffusion equations with  $(p, q)$ -Laplacian*, Comm. Pure Appl. Math., **4** (2005), 9–22.
- [7] P. C. Fife, *Mathematical aspects of reacting and diffusing systems*, Lecture Notes in Biomathematics, Vol. 28, Springer-Verlag, Berlin, 1979.
- [8] L. Gasiński and N. S. Papageorgiou, *Asymmetric  $(p, 2)$ -equations with double resonance*, Calc. Var. Partial Differential Equations, **56** (2017), 1–23.
- [9] L. Jeanjean, *Local conditions insuring bifurcation from the continuous spectrum*, Math. Z., **232** (1999), 651–664.
- [10] P. Lindqvist, *On the equation  $\operatorname{div}(|\nabla u|^{p-2}\nabla u) + \lambda|u|^{p-2}u = 0$* , Proc. Amer. Math. Soc., **109** (1990), 157–164.
- [11] N. S. Papageorgiou and V. D. Rădulescu, *Qualitative phenomena for some classes of quasilinear elliptic equations with multiple resonance*, Appl. Math. Optim., **69** (2014), 393–430.

---

The authors thank the referee for his helpful comments and suggestions.

- [12] N. S. Papageorgiou, V. D. Rădulescu and D.D. Repovš, *Existence and multiplicity of solutions for resonant  $(p, 2)$ -equations*, Adv. Nonlinear Stud., **18** (2018), 105–129.
- [13] M. Struwe, *Variational methods: Applications to nonlinear partial differential equations and Hamiltonian systems*, 4th edition, A Series of Modern Surveys in Mathematics, Vol. 34, Springer, Berlin, 2008.
- [14] A. Zerouali, B. Karim, O. Chakrone and A. Boukhsas, *Resonant Steklov eigenvalue problem involving the  $(p; q)$ -Laplacian*, Afr. Mat., **30** (2019), 171–179.
- [15] A. Zerouali, B. Karim, O. Chakrone and A. Boukhsas, *On a positive solution for  $(p; q)$ -Laplace equation with nonlinear boundary conditions and indefinite weights*, Bol. Soc. Parana. Mat. (3), **38** (2020), 219–233.
- [16] V. V. E. Zhikov, *Averaging of functionals of the calculus of variations and elasticity theory*, Math. USSR-Izv., **29** (1987), 33–66.

Received December 24, 2020  
 Accepted May 4, 2021

*University Moulay Ismail of Meknes*  
*FST Errachidia*  
*LMIMA Laboratory, ROLALI Group*  
*Errachidia, Morocco*  
*E-mail: abdelmajidboukhsas@gmail.com*  
<https://orcid.org/0000-0002-9317-8232>

*Regional Centre of Trades Education and Training*  
*Department of Mathematics*  
*Oujda, Morocco*  
*E-mail: abdellahzerouali@yahoo.fr*  
<https://orcid.org/0000-0001-9090-4094>

*University Mohammed First*  
*Faculty of Sciences*  
*Department of Mathematics*  
*Oujda, Morocco*  
*E-mail: chakrone@yahoo.fr*  
<https://orcid.org/0000-0002-2208-4220>

*University Moulay Ismail of Meknes*  
*FST Errachidia*  
*Department of Mathematics*  
*Errachidia, Morocco*  
*E-mail: karembelf@gmail.com*  
<https://orcid.org/0000-0002-7455-5434>