INTEGRAL SOLUTION OF A CONFORMABLE FRACTIONAL INTEGRO-DIFFERENTIAL EQUATION WITH NONLOCAL CONDITION

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Abstract. This paper deals with the existence and uniqueness of the integral solution of a nondense integro-differential equation with nonlocal condition in the frame of conformable fractional derivative. The main results are obtained by using some fixed point theorems combined with a integrated semigroup approach.

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