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ON FLAT EPIMORPHISMS OF RINGS AND POINTWISE LOCALIZATIONS

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Abstract. In this paper all rings are commutative. We prove some new results on flat epimorphisms of rings and pointwise localizations. Especially among them, it is proved that a ring R is an absolutely flat (von-Neumann regular) ring if and only if it is isomorphic to the pointwise localization $R^{(-1)}R$, or equivalently, each R-algebra is R-flat. For a given minimal prime ideal $\mathfrak{p}$ of a ring R, the surjectivity of the canonical map $R \to R_{\mathfrak{p}}$ is characterized. Finally, we give a new proof to the fact that in a flat epimorphism of rings, the contraction-extension of an ideal equals the same ideal.

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Key words. Absolutely flat ring, flat epimorphism, pointwise localiation.

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