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FINITE p-GROUPS WHICH ARE NON-INNER NILPOTENT

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Abstract. A group G is called a non-inner nilpotent group, whenever it is nilpotent with respect to a non-inner automorphism. In 2018, all finitely generated abelian non-inner nilpotent groups have been classified. Actually, the authors proved that a finitely generated abelian group G is a non-inner nilpotent group, if G is not isomorphic to cyclic groups $\mathbb{Z}_{p_1p_2\dots p_t}$ and $\mathbb{Z}$, for a positive integer t and distinct primes $p_1, p_2, \dots, p_t$. In this paper, we make this conjecture that all finite non-abelian p-groups are non-inner nilpotent and we prove this conjecture for finite p-groups of nilpotency class 2 or of co-class 2.

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