

ON THE SOLVABILITY OF A SYSTEM OF
CAPUTO-HADAMARD FRACTIONAL HYBRID DIFFERENTIAL
EQUATIONS SUBJECT TO SOME HYBRID BOUNDARY
CONDITIONS

ABDELATIF BOUTIARA, MAAMAR BENBACHIR, and KADDOUR GUERBATI

Abstract. In this work, we give some existence and regularity results for a system of a new class of hybrid Caputo-Hadamard fractional differential equations under hybrid boundary conditions. The technique of investigation is essentially based on the use of a well known hybrid fixed point theorem.

MSC 2010. Primary 26A33; Secondary 34B25, 34B15.

Key words. Hybrid fractional differential equation, Caputo-Hadamard derivative, hybrid boundary value problem, fixed point theorems.

REFERENCES

- [1] B. Ahmad, S.K. Ntouyas and J. Tariboon, *A nonlocal hybrid boundary value problem of Caputo fractional integro-differential equations*, Acta Math. Sci., **36** (2016), 1631–1640.
- [2] B. Ahmad and S.K. Ntouyas, *A note on fractional differential equations with fractional separated boundary conditions*, Abstr. Appl. Anal., **2012** (2012), Article 818703, 1–11.
- [3] A. Boutiara, M.S. Abdo and M. Benbachir, *Existence results for ψ -Caputo fractional neutral functional integro-differential equations with finite delay*, Turkish J. Math., **44** (2020), 2380–2401.
- [4] A. Boutiara, M. Benbachir and K. Guerbati, *Hilfer fractional hybrid differential equations with multi-point boundary hybrid conditions*, Inter. J. Modern Math. Sci., **19** (2021), 17–33.
- [5] A. Boutiara, M. Benbachir and K. Guerbati, *Existence and uniqueness solutions of a BVP for nonlinear Caputo-Hadamard fractional differential equation*, J. Appl. Nonlinear Dyn., **11** (2022), 359–374.
- [6] A. Boutiara, K. Guerbati and M. Benbachir, *Caputo-Hadamard fractional differential equation with three-point boundary conditions in Banach spaces*, AIMS Math., **5** (2020), 259–272.
- [7] A. Boutiara, M. Benbachir and K. Guerbati, *Caputo type fractional differential equation with nonlocal Erdélyi-Kober type integral boundary conditions in Banach spaces*, Surv. Math. Appl., **15** (2020), 399–418.
- [8] A. Boutiara, M. Benbachir and K. Guerbati, *Measure of noncompactness for nonlinear Hilfer fractional differential equation in Banach spaces*, Ikonion J. Math., **1** (2019), 55–67.

The authors thank the referee for his helpful comments and suggestions.

- [9] C. Derbazi, H. Hammouche, M. Benchohra and Y. Zhou, *Fractional hybrid differential equations with three-point boundary hybrid conditions*, Adv. Difference Equ., **125** (2019), 1–11.
- [10] B.C. Dhage, *A fixed point theorem in Banach algebras with applications to functional integral equations*, Kyungpook Math J., **44** (2004), 145–155.
- [11] B.C. Dhage, *Quadratic perturbations of periodic boundary value problems of second order ordinary differential equations*, Differ. Equ. Appl., **2** (2010), 465–486.
- [12] B.C. Dhage, *Basic results in the theory of hybrid differential equations with mixed perturbations of second type*, Funct. Differ. Equ., **19** (2012), 1–20.
- [13] A.E.M. Herzallah and D. Baleanu, *On fractional order hybrid differential equations*, Abstr. Appl. Anal., **2014** (2014), Article 389386, 1–7.
- [14] K. Hilal and A. Kajouni, *Boundary value problems for hybrid differential equations with fractional order*, Adv. Difference Equ., **2015** (2015), Article 183, 1–19.
- [15] Y.Y. Gambo, F. Jarad, D. Baleanu and T. Abdeljawad, *On Caputo modification of the Hadamard fractional derivatives*, Adv. Difference Equ., **2014** (2014), 1–12.
- [16] F. Jarad, D. Baleanu and A. Abdeljawad, *Caputo-type modification of the Hadamard fractional derivatives*, Adv. Difference Equ., **142** (2012), 1–8.
- [17] V. Kac and P. Cheung, *Quantum Calculus*, Springer, New York, 2002.
- [18] A.A. Kilbas, H.H. Srivastava and J.J. Trujillo, *Theory and applications of fractional differential equations*, Elsevier, Amsterdam, 2006.
- [19] I. Podlubny, *Fractional differential equations*, Academic Press, 1999.
- [20] H. Rebai and D. Seba, *Weak solutions for nonlinear fractional differential equation with fractional separated boundary conditions in banach spaces*, Filomat, **32** (2018), 1117–1125.
- [21] S. G. Samko, A.A. Kilbas and O.I. Marichev, *Fractional Integrals and Derivatives Theory and Applications*, Gordon and Breach Science Publishers, Switzerland, 1993.
- [22] M.E. Samei, V. Hedayati and S. Rezapour, *Existence results for a fraction hybrid differential inclusion with Caputo-Hadamard type fractional derivative*, Adv. Difference Equ. **163** (2019), 1–15.
- [23] S. Sitho, S.K. Ntouyas and J. Tariboon, *Existence results for hybrid fractional integro-differential equations*, Bound. Value Probl., **2015** (2015), Article 113, 1–13.
- [24] S. Sun, Y. Zhao, Z. Han and Y. Lin, *The existence of solutions for boundary value problem of fractional hybrid differential equations*, Commun. Nonlinear Sci. Numer. Simul., **17** (2012), 4961–4967.
- [25] A. Yacine and B. Nouredine, *Boundary value problem for Caputo-Hadamard fractional differential equations*, Surv. Math. Appl., **12** (2017), 103–115.
- [26] W. Yukunthorn, B. Ahmad, K.S. Ntouyas and J. Tariboon, *On Caputo-Hadamard type fractional impulsive hybrid systems with nonlinear fractional integral conditions*, Nonlinear Anal. Hybrid Syst., **19** (2016), 77–92.
- [27] Y. Zhao, S. Sun, Z. Han and Q. Li, *Theory of fractional hybrid differential equations*, Computers and Mathematics with Applications, **62** (2011), 1312–1324.

Received March 22, 2020

Accepted February 12, 2021

University of Ghardaia
Laboratory of Mathematics and Applied Sciences
Ghardaia, Algeria
E-mail: boutiara.a@yahoo.com
E-mail: guerbati.k@yahoo.com

Saad Dahlab University
Faculty of Sciences
Blida, Algeria
E-mail: mbenbachir2001@gmail.com