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A COMMON PROPERTY TO GENERALIZED CONTRACTIONS AND EXISTENCE OF FIXED POINTS

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Abstract. The definitions of several types of generalized contractions for an application T from a complete metric space (X, d) in itself given in [14] are recalled and reviewed. A common property to all these concepts is put in light, namely: there exists $\alpha > 0$ such that, for all $x \in X$, $x \neq T(x)$, there exists $u \in X \setminus \{x\}$ satisfying:

$$[H] \qquad \mathrm{d}(u, T(u)) + \alpha \mathrm{d}(x, u) \leq \mathrm{d}(x, T(x)).$$

We observe that assumption $[H]$ is fulfilled in most cases treated in [14] and we prove that assumption $[H]$ and lower semi-continuity of the function $x \longmapsto \mathrm{d}(x, T(x))$ ensure existence of a fixed point along with a sharp estimate for the distance to the fixed-points set.

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Key words. Ekeland's variational principle, fixed point, lower semicontinuty, generalized contractions.

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