

WELL-POSEDNESS AND GENERAL ENERGY DECAY OF  
SOLUTIONS FOR A PETROVSKY EQUATION WITH A  
NONLINEAR STRONG DISSIPATION

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**Abstract.** In this paper we consider a nonlinear Petrovsky equation in a bounded domain with a strong dissipation

$$u'' + \Delta^2 u - g(\Delta u') = 0.$$

and prove the existence and the uniqueness of the solution using the energy method combined with the Faedo-Galerkin procedure under certain assumptions for  $g$ . Furthermore, we study the asymptotic behaviour of the solutions using a perturbed energy method.

**MSC 2010.** 35B40, 35B45, 35L70.

**Key words.** Well-posedness, general decay, multiplier method, convexity, Petrovsky equation.

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