

UNIQUENESS AND EXISTENCE OF SOLUTIONS FOR
NONLINEAR FRACTIONAL DIFFERENTIAL EQUATIONS WITH
TWO FRACTIONAL ORDERS

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Abstract. In this work, we study the existence and uniqueness of solutions for integro-differential equations involving two fractional orders. By using the Banach's fixed point theorem, Leray-Schauder's nonlinear alternative and Leray-Schauder's degree theory, the existence and uniqueness of solutions are obtained. Some illustrative examples are also presented.

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REFERENCES

- [1] C.Z. Bai and J.X. Fang, *The existence of a positive solution for a singular coupled system of nonlinear fractional differential equations*, Appl. Math. Comput., **150** (2004), 611–621.
- [2] Z. Bai and H. Lu, *Positive solutions for boundary value problem of nonlinear fractional differential equation*, J. Math. Anal. Appl., **311** (2005), 495–505.
- [3] Z. Cui, P. Yu and Z. Mao, *Existence of solutions for nonlocal boundary value problems of nonlinear fractional differential equations*, Adv. Dyn. Syst. Appl., **7** (2012), 31–40.
- [4] D. Delbosco and L. Rodino, *Existence and uniqueness for a nonlinear fractional differential equation*, J. Math. Anal. Appl., **204** (1996), 429–440.
- [5] K. Diethelm and N.J. Ford, *Analysis of fractional differential equations*, J. Math. Anal. Appl., **265** (2002), 229–248.
- [6] A.M.A. El-Sayed, *Nonlinear functional differential equations of arbitrary orders*, Nonlinear Anal., **33** (1998), 181–186.
- [7] V. Gaychuk, B. Datsko and V. Meleshko, *Mathematical modeling of time fractional reaction diffusion systems*, J. Comput. Appl. Math., **220** (2008), 215–225.
- [8] R. Hilfer, *Applications of fractional calculus in physics*, World Scientific, River Edge, NJ, USA, 2000.
- [9] M. Houas and Z. Dahmani, *New results for a Caputo boundary value problem*, American Journal of Computational and Applied Mathematics, **3** (2013), 143–161.
- [10] M. Houas, Z. Dahmani and M. Benbachir, *New results for a boundary value problem for differential equations of arbitrary order*, International Journal of Modern Mathematical Sciences, **7** (2013), 195–211.

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- [11] M. Houas and Z. Dahmani, *On existence of solutions for fractional differential equations with nonlocal multi-point boundary conditions*, Lobachevskii J. Math., **37** (2016), 120–128.
- [12] M. Houas and M. Benbachir, *Existence and uniqueness results for nonlinear differential equations of arbitrary order*, International Journal of Nonlinear Analysis and Applications, **6** (2015), 77–92.
- [13] M. Houas, *Existence of solutions for fractional differential equations involving two Riemann-Liouville fractional orders*, Anal. Theory Appl., **34** (2018), 253–274.
- [14] M. Houas and M. Benbachir, *Existence solutions for four point boundary value problems for fractional differential equations*, Pure and Applied Mathematics Letters, **2015** (2015), 37–49.
- [15] M. Houas, *Solvability of a system of fractional hybrid differential equations*, Communications in Optimization Theory, **2018**, Article 12, 1–9.
- [16] A.A. Kilbas and S.A. Marzan, *Nonlinear differential equation with the Caputo fraction derivative in the space of continuously differentiable functions*, Differ. Equ., **41** (2005), 84–89.
- [17] V. Lakshmikantham and A.S. Vatsala, *Basic theory of fractional differential equations*, Nonlinear Anal., **69** (2008), 2677–2682.
- [18] F. Mainardi, *Fractional calculus – some basic problem in continuum and statistical mechanics. Fractals and fractional calculus in continuum mechanics*, Springer, Vienna, 1997, pp. 291–348.
- [19] A. Saadi and M. Benbachir, *Positive solutions for three-point nonlinear fractional boundary value problems*, Electron. J. Qual. Theory Differ. Equ., **2** (2011), 1–19.
- [20] C. Thaiprayoon, S.K. Ntouyas and J. Tariboon, *On the nonlocal Katugampola fractional integral conditions for fractional Langevin equation*, Adv. Difference Equ., **374** (2015), 1–16.
- [21] W. Yukunthorn, S.K. Ntouyas and J. Tariboon, *Nonlinear fractional Caputo-Langevin equation with nonlocal Riemann-Liouville fractional integral conditions*, Adv. Difference Equ., **315** (2014), 1–18.

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