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DIRICHLET BOUNDARY VALUE PROBLEM RELATED TO THE $p(x)$-LAPLACIAN WITH DISCONTINUOUS NONLINEARITY

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Abstract. In this paper, we prove the existence of a weak solution for the Dirichlet boundary value problem related to the $p(x)$-Laplacian

$$-\mathrm{div}(|\nabla u|^{p(x)-2}\nabla u) + u \in -[\underline{g}(x,u), \overline{g}(x,u)],$$

by using the degree theory after turning the problem into a Hammerstein equation. The right hand side g is a possibly discontinuous function in the second variable satisfying some non-standard growth conditions.

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Key words. Set-valued operators, nonlinear elliptic equation, $p(x)$-Laplacian, Sobolev spaces with variable exponent, degree theory.

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