BOUNDARY VALUE PROBLEMS FOR HILFER FRACTIONAL DIFFERENTIAL EQUATIONS WITH KATUGAMPOLA FRACTIONAL INTEGRAL AND ANTI-PERIODIC CONDITIONS

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Abstract. The purpose of this paper is to investigate the existence and uniqueness of solutions for a new class of nonlinear fractional differential equations involving Hilfer fractional operator with fractional integral boundary conditions. Our analysis relies on classical fixed point theorems and the Boyd-Wong nonlinear contraction. At the end, an illustrative example is presented. The boundary conditions introduced in this work are of quite general nature and can be reduce to many special cases by fixing the parameters involved in the conditions.

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Key words. Fractional differential equation, Hilfer fractional derivative, Katugampola fractional integral, fixed point theorems.

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