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A GENERALIZATION OF WEIGHTED BILINEAR HARDY INEQUALITY

BOUHARKET BENAISSA and MEHMET ZEKI SARIKAYA

Abstract. In this paper, we give some new generalizations of the weighted bilinear Hardy inequality by using weighted mean operators $S := (Sf)_g^w$, where f nonnegative integrable function with two variables on $\triangle = (0, +\infty) \times (0, +\infty)$, defined by

$$S(x, y) = \int_a^x \int_c^y \frac{w(t)w(s)}{W(t)W(s)} g(f(t, s))\mathrm{d}s\mathrm{d}t,$$

with

$$W(z) = \int_0^z w(r)\mathrm{d}r, \quad for\ z \in (0, +\infty),$$

where w is a weight function and g is a nonnegative continuous function on $(0, +\infty)$.

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Key words. Hölder's inequality, Hardy-Type Integral Inequality, weight function.

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University of Tiaret, Algeria
Faculty of Material Sciences
Laboratory of Informatics and Mathematics
E-mail: bouharket.benaissa@univ-tiaret.dz
https://orcid.org/0000-0002-1195-6169

Düzce University, Düzce, Turkey
Faculty of Science and Arts
Department of Mathematics
E-mail: sarikayamz@gmail.com
https://orcid.org/0000-0002-6165-9242