

A NOTE ON THE DIOPHANTINE EQUATION

$$x^2 - kxy + ky^2 + ly = 0$$

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**Abstract.** We investigate the Diophantine equation  $x^2 - kxy + ky^2 + ly = 0$  for integers  $k$  and  $l$  with  $k$  even. We give a characterization of the positive solutions of this equation in terms of  $k$  and  $l$ . We also consider the same equation when  $l = p^n$  and  $k \equiv 2 \pmod{p}$  for  $p \equiv 3 \pmod{4}$ ;  $l = 2^r 3^s$  and  $k = 2k' + 1$  with  $k' \equiv 2 \pmod{3}$  where  $n, s, t$  are non-negative integers.

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