

KERNEL STABLE AND UNIQUELY GENERATED MODULES

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Abstract. Module theoretic notion of annihilator-stable rings is defined and some characterizations of it are studied in the present paper. M is called a kernel-stable module if every element $\alpha \in \text{End}(M)$ satisfies the following condition: if $\alpha(M) + \text{Ker}\beta = M$, $\beta \in \text{End}(M)$, then $(\alpha - \gamma)(m) \in \text{Ker}\beta$ for an automorphism γ of M and for all $m \in M$. For a pseudo-semi-projective module M , this notion is equivalent to the uniquely generated module which was defined in [9].

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