WELL-POSEDNESS AND EXPONENTIAL DECAY FOR A LAMINATED BEAM IN THERMOELASTICITY OF TYPE III WITH DELAY TERM

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Abstract. In this article, we study the well-posedness and asymptotic behaviour of solutions to a laminated beam in thermoelasticity of type III with delay term in the first equation. We show that the system is well-posed by using Lumer-Philips theorem and prove that the system is exponentially stable if and only if the wave speeds are equal.

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