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GLOBAL EXISTENCE AND ENERGY DECAY OF SOLUTIONS FOR A WAVE EQUATION WITH A TIME-VARYING DELAY TERM

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Abstract. In this paper, we consider in a bounded domain the wave equation with a weak internal time-varying delay term:

$$u_{tt}(x,t) - \Delta_x u(x,t) + \mu_1(t)\, u_t(x,t) + \mu_2(t)\, u_t(x, t-\tau(t)) = 0.$$

Under appropriate conditions on the functions μ_1 and μ_2, we prove global existence of solutions by the Faedo-Galerkin method and establish a decay rate estimate for the energy using the multiplier method.

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Key words. Wave equation, delay term, decay rate, multiplier method.

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