MATHEMATICA, Tome 62 (85), N° 2, 2020, pp. 148–159

GENERALIZED SEMIDERIVATIONS IN PRIME RINGS WITH ALGEBRAIC IDENTITIES

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Abstract. Let $\mathcal{R}$ be a prime ring with center $Z(\mathcal{R})$. Suppose that $\mathcal{R}$ admits a generalized semiderivation F with associated derivation $d \neq 0$. In the present paper we investigate the commutativity of a prime ring $\mathcal{R}$ satisfying any one of the identities: (**i**) $F([x,y]) \in Z(\mathcal{R})$, (**ii**) $F(x \circ y) \in Z(\mathcal{R})$, (**iii**) $F(xy) \pm xy \in Z(\mathcal{R})$, (**iv**) $F(xy) \pm yx \in Z(\mathcal{R})$, (**v**) $[F(x), F(y)] = 0$, (**vi**) $F(x) \circ F(y) = 0$ for all $x, y \in \mathcal{R}$.

MSC 2010. 16U80, 16W25.

Key words. Involution, prime ring, generalized derivation, generalized semiderivation, commutativity, differential identities.

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The authors thank the referee for his helpful comments and suggestions.

DOI: 10.24193/mathcluj.2020.2.04

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Received March 23, 2019
Accepted August 24, 2019

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