PERIODICITY AND POSITIVITY IN NONLINEAR NEUTRAL INTEGRO-DYNAMIC EQUATIONS WITH VARIABLE DELAY

MALIK BELAID, ABDELOUAHEB ARDJOUNI, and AHCENE DJOUDI

Abstract. Let $\mathbb{T}$ be a periodic time scale. We use Krasnoselskii's fixed point theorem for a sum of two operators to show new results on the existence of periodic and positive periodic solutions of the nonlinear neutral integro-dynamic equation with variable delay of the form

$$x^{\triangle}(t) = -\int_{t-\tau(t)}^{t} a(t,s)\,x(s)\,\Delta s + Q(t, x(t-\tau(t)))^{\triangle}$$
$$+\, G(t, x(t), x(t-\tau(t))),\ t\ \in\ \mathbb{T}.$$

We invert this equation to construct a sum of a contraction and a completely continuous map which is suitable for applying Krasnoselskii's theorem. The uniqueness results of this equation are studied by the contraction mapping principle. The results obtained here extend the work of Mesmouli, Ardjouni and Djoudi [15].

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University of Annaba
Department of Mathematics
P.O. Box 12, Annaba, Algeria
E-mail: `malikbelaid51@gmail.com`

University of Souk Ahras
Department of Mathematics and Informatics
P.O. Box 1553, Souk Ahras, Algeria
E-mail: `malikbelaid51@gmail.com`

University of Annaba
Department of Mathematics
P.O. Box 12, Annaba, Algeria
E-mail: `adjoudi@yahoo.com`