

ON THE POLYNOMIAL SOLUTIONS OF GENERAL  
POLYNOMIAL DIFFERENTIAL EQUATIONS

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**Abstract.** We deal with the ordinary differential equation of the form  $y^m dy/dx = P(x, y)$  where  $m \geq 2$  and  $P(x, y)$  is a real polynomial in the variables  $x$  and  $y$  of degree  $n$  in the variable  $y$ . We study the maximum number of the polynomial solutions of this equation with respect to  $n$ . We also consider the multiplicity of polynomial limit cycles.

**MSC 2010.** 37D99.

**Key words.** Polynomial ordinary differential equations, polynomial solutions.

REFERENCES

- [1] D. Behoul and S.S. Cheng, *Computation of all polynomial solutions of a class of non-linear differential equations*, Computing, **77** (2006), 163–177.
- [2] J.G. Campbell and M. Golomb, *On the polynomial solutions of a Riccati equation*, Amer. Math. Monthly, **61** (1954), 402–404.
- [3] J. Giné, M. Grau and J. Llibre, *On the polynomial limit cycles of polynomial differential equations*, Israel J. Math., **181** (2011), 461–475.
- [4] N.G. Lloyd, *The number of periodic solutions of the equation  $\dot{z} = z^N + p_1(t)z^{N-1} + \dots + p_N(t)$* , Proc. Lond. Math. Soc., **27** (1973), 667–700.
- [5] N.G. Lloyd, *A note on the number of limit cycles of certain two-dimensional systems*, J. Lond. Math. Soc., **20** (1979), 277–286.
- [6] E.D. Rainville, *Necessary conditions for polynomial solutions of certain Riccati equations*, Amer. Math. Monthly, **43** (1936), 473–476.
- [7] C. Valls, *On the polynomial solutions of some generalized polynomial differential equations*, preprint.

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