

COURNOT EQUILIBRIUM  
IN CASE OF (-1)-CONCAVE PRICE FUNCTION

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**Abstract.** We consider a class of homogeneous Cournot oligopolies with  $(-1)$ -concave price function. We show some useful properties of the revenue function in case of  $(-1)$ -concave price function and prove the existence of an equilibrium in the continuous and non-differentiable case. A simple proof of an equilibrium uniqueness result in the smooth case with  $(-1)/N$ -concave ( $N$ -number of the firms in the market) price function is provided.

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**Key words.** Cournot games, generalized concavity, price function, pure-strategy Nash equilibrium.

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