OSCILLATION ANALYSIS FOR NONLINEAR NEUTRAL DIFFERENTIAL EQUATIONS OF SECOND ORDER WITH SEVERAL DELAYS AND FORCING TERM

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Abstract. In this paper, sufficient conditions are obtained for the oscillation of the nonlinear neutral forced differential equations of second-order with several delays of the form

(E)

$$\frac{\mathrm{d}}{\mathrm{d}t}\left[r(t)\frac{\mathrm{d}}{\mathrm{d}t}\big[x(t) + p(t)x(t-\tau)\big]\right] + \sum_{i=1}^{m} q_i(t)H\big(x(t-\sigma_i)\big) = f(t), \quad t \geq t_0 > 0,$$

under the assumptions $\int^{\infty} \frac{1}{r(\eta)}\mathrm{d}\eta = \infty$ and $\int^{\infty} \frac{1}{r(\eta)}\mathrm{d}\eta < \infty$ for various ranges of the bounded neutral coefficient p. Also, an attempt is made to discuss existence of bounded positive solutions of (E). Further, one illustrative example showing the applicability of the new results is included.

MSC 2010. 34C10, 34C15, 34K40.

Key words. Contraction principle, delay, existence of positive solution, neutral differential equations, non-linear, nonoscillation, oscillation.

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