

SOME PROPERTIES OF THE RESOLVENT OF  
STURM-LIOUVILLE OPERATORS  
ON UNBOUNDED TIME SCALES

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**Abstract.** In this article, we investigate the resolvent operator of Sturm-Liouville problem on unbounded time scales. We obtain integral representations for the resolvent of this operator. Later, we discuss some properties of the resolvent operator, such as Hilbert-Schmidt's kernel property and compactness. Finally, we give a formula for the Titchmarsh-Weyl function of the Sturm-Liouville problem on unbounded time scales.

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**Key words.** Hilbert-Schmidt kernel, resolvent operator, singular point, spectral function, Sturm-Liouville operator, time scale, Titchmarsh-Weyl function.

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