

## $UJ$ -ENDOMORPHISM RINGS

SERAP ŞAHİNKAYA and TULAY YILDIRIM

**Abstract.** In this paper, we introduce and study  $UJ$ -modules, that is modules  $M$  for which their endomorphism rings  $E_M$  are right  $UJ$ . We show, in particular, that: (1) if  $M$  is a left  $UJ$ -module over a ring  $R$ , then  $M$  is Dedekind finite; (2)  $M$  is a  $UJ$ -module iff all clean elements of  $E_M$  are  $J$ -clean; (3)  $M$  is a clean  $UJ$ -module iff  $E_M/J(E_M)$  is a Boolean ring and the idempotents lift modulo  $J(E_M)$  (equivalently,  $M$  is a  $J$ -clean module); and (4)  $M$  is a clean  $UJ$ -module such that  $J(E_M)$  is nil iff  $M$  is a conjugate nil clean  $UJ$ -module. We also give characterizations of the trivial extension and the (trivial) Morita context,  $R[x]/(x^2)$  and the tail rings which are right  $UJ$ .

**MSC 2010.** 30C45.

**Key words.** Unit, radical, clean module and ring, conjugate nil clean module and ring,  $UJ$ -module and ring.

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*Gebze Technical University*  
*Department of Mathematics*  
*Gebze, Turkey*  
*E-mail: srpsahinkaya@gmail.com*  
*E-mail: tyildirim@gtu.edu.tr*