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MIXED BOUNDARY VALUE PROBLEMS FOR THE STOKES SYSTEM ON COMPACT RIEMANNIAN MANIFOLDS

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Abstract. The purpose of this paper is to show a well-posedness result for a Dirichlet-Neumann boundary value problem for the Stokes system on compact Riemannian manifolds. Using layer potential techniques, we derive an equivalent boundary integral system for the Stokes system and prove the invertibility of the related matrix integral operator.

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Key words. Stokes system, mixed problem, layer potential analysis, compact Riemannian manifolds.

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