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ON THE DIOPHANTINE EQUATION $x^5 + ky^3 = z^5 + kw^3$

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Abstract. In this article we consider the symmetric Diophantine equation $x^m + ky^n = z^m + kw^n$, where k is a rational number and prove that, for any rational number k, the equation $x^5+ky^3 = z^5+kw^3$ has infinitely many rational nontrivial solutions. The strategy is to use the elliptic fibration method.

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Key words. Diophantine equation, elliptic curve.

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