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ON THE CONCEPT OF φ-ENTROPY

MEHDI RAHIMI

Abstract. In this paper, the concept of φ-entropy is defined and some of the its properties are proved. It is a type of generalized entropy with generalized properties. It is invariant under topological conjugacy and satisfies a generalized version of Jaccob's Theorem. Finally, we will extract the Kolmogorov entropy as a special case, by setting φ to be the identity function.

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University of Qom
Faculty of Science
Department of Mathematics
Qom, Iran
E-mail: m10.rahimi@gmail.com