# A BRIEF REMARK ON BALANCING-WIEFERICH PRIMES

UTKAL KESHARI DUTTA, BIJAN KUMAR PATEL, and PRASANTA KUMAR RAY

**Abstract.** A prime $p$ is said to be a balancing-Wieferich prime if it satisfies the congruence $B_{p-(\frac{8}{p})} \equiv 0 \pmod{p^2}$, equivalently $\pi(p) = \pi(p^2)$. Here $B_n$ denotes the $n$-th balancing number and $\pi(m)$ is the period of balancing numbers modulo any positive integer $m$. In this note, we establish some conditions related to the balancing-Wieferich primes.

**MSC 2010.** 11B25, 11B39, 11B41.

**Key words.** Balancing numbers, Wieferich primes, balancing-Wieferich primes, periodicity.

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*Sambalpur University*
*Department of Mathematics*
*Sambalpur, India*
*E-mail:* `utkaldutta@gmail.com`

*International Institute of Information Technology*
*Bhubaneswar, India*
*E-mail:* `iiit.bijan@gmail.com`

*Sambalpur University*
*Department of Mathematics*
*Sambalpur, India*
*E-mail:* `prasantamath@suniv.ac.in`