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# REFINING LAH-RIBARIĆ INTEGRAL INEQUALITY FOR DIVISIONS OF MEASURABLE SPACE

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**Abstract.** In this paper, we establish some refinements of Lah-Ribarić inequality for the general Lebesgue integral on divisions of measurable space. Applications for discrete inequalities and weighted means of positive numbers are also given. Some examples related to Hermite-Hadamard inequality for convex functions are provided as well.

**MSC 2010.** Primary 26D15; Secondary 26D10.

**Key words.** Jensen's inequality, convex functions, Lebesgue integral, weighted means, Lah-Ribarić inequality, special means.

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