EXISTENCE OF SOLUTION FOR THIRD-ORDER THREE-POINT BOUNDARY VALUE PROBLEM

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Abstract. By imposing some conditions on the nonlinear term f, we construct a lower solution and an upper solution to prove the existence of a solution for a type of nonlinear third-order nonlocal boundary value problem. Our main tools are the upper and the lower solution method and the Schauder fixed point theorem. The results are illustrated by an example.

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REFERENCES

[1] D.R. Anderson, *Existence of solution for some third-order boundary value problems*, Electron. J. Differential Equations, **25** (2008), 1–6.

[2] N. Bouteraa and S. Benaicha, *Triple positive solutions of higher-order nonlinear boundary value problem*, Journal of Computer Science and Computational Mathematics, **7** (2017), 25–30.

[3] Z.B. Bai, *Green's function for a third-order generalized right focal problem*, J. Math. Anal. Appl., **288** (2003), 1–14.

[4] S. Clark and J. Henderson, *Uniqueness implies existence and uniqueness criterion for a nonlocal boundary value problems for third-order differential equations*, Proc. Amer. Math. Soc., **134** (2006), 3363–3372.

[5] K. Dimling, *Multivalued differential equations*, Walter De Gruyter, Berlin, 1982.

[6] Y. Feng and S. Liu, *Solvability of a third-order two-point boundary value problem*, Appl. Math. Lett., **18** (2005), 1034–1040.

[7] M. Gregus, *Third order linear differential equations*, D. Reidel Publishing Company, Dordrecht, 1987.

[8] C.P. Gupta, *A generalized multi-point boundary value problem for second-order ordinary differential equations*, Appl. Math. Comput., **89** (1998), 133–146.

[9] M.R. Grossinho and F.M. Minohs, *Existence result for some third-order separated boundary value problems*, Nonlinear Anal., **47** (2004), 2407–2418.

[10] L.J. Guo, J.P. Sun and Y.H. Zhao, *Existence of positive solution for nonlinear third-order three-point boundary value problem*, Nonlinear Anal., **68** (2008), 3151–3158.

[11] F. Haddouchi and S. Benaicha, *Multiple positive solutions for a nonlinear three-point integral boundary value proble*, International Journal of Open Problems in Computer Science and Mathematics, **8** (2015), 29–42.

[12] F. Haddouchi and S. Benaicha, *Positive solutions of a nonlinear three-point eigenvalue problem with integral boundary value conditions*, Rom. J. Math. Comput. Sci., **5** (2015), 202–213.

[13] R. Ma, *A survey on nonlocal boundary value problems*, Appl. Math. E-Notes, **7** (2007), 257–279.

[14] A. Rezaigia and S. Kelaiaia, *Existence of positive solution for a third-order three-point boundary value problem*, Mat. Vesnik, **68** (2016), 12–25.

[15] S. Reich and D. Shoikhet, *Nonlinear semi group, fixed points and geometry of domains in Banach spaces*, Imperial College Press, London, 2005.

[16] L. Shuhong, *Positive solutions of nonlinear singular third-order two-point boundary-value problem*, J. Math. Anal. Appl., **323** (2006), 413–425.

[17] J.P. Sun and H.E. Zhang, *Existence of solutions to third-order m-point boundary value problems*, Electron. J. Differential Equations, **125** (2008), 1–9.

[18] Y. Sun, *Positive solutions for third-order three-point nonhomogeneous boundary value problems*, Appl. Math. Lett., **22** (2009), 45–51.

[19] F.J. Torres, *Positive solutions for a third-order three-point boundary-value problem.*, Electron. J. Differential Equations, **147** (2013), 1–11.

[20] Q. Yao, *The existence and multiplicity of positive solutions for a third-order three-point regular boundary value problem*, Acta Math. Appl. Sinica, **19** (2003), 117–122.

[21] Q. Yao, *Positive solutions of a weak semipositone third-order three-point boundary value problem*, Journal of Mathematical Research & Exposition, **30** (2010), 173–180.

[22] L. Zhang, B. Sun and C. Xing, *Existence of solutions to a third-order three-point boundary value problem*, Annals of Differential Equations, **3** (2012), 368–372.

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