

OSCILLATION ANALYSIS FOR NONLINEAR NEUTRAL  
DIFFERENTIAL EQUATIONS OF SECOND ORDER  
WITH SEVERAL DELAYS

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**Abstract.** In this work, oscillatory and asymptotic behavior of solutions of a class of nonlinear neutral differential equations of second-order with several delays of the form

$$(E) \quad \frac{d}{dt} \left[ a(t) \frac{d}{dt} [x(t) + p(t)x(t - \tau)] \right] + \sum_{i=1}^m q_i(t)H(x(t - \sigma_i)) = 0, \quad t \geq t_0,$$

are studied, for various ranges of the bounded neutral coefficient  $p$ , under the assumptions  $\int_0^\infty \frac{d\eta}{a(\eta)} = \infty$  and  $\int_0^\infty \frac{d\eta}{a(\eta)} < \infty$ . Also, an attempt is made to discuss existence of bounded positive solutions of (E). Further, some illustrative examples, showing the applicability of the new results, are included.

**MSC 2010.** 34C10, 34C15, 34K40.

**Key words.** Oscillation, nonoscillation, nonlinear, delay, neutral differential equations, existence of positive solution, contraction principle.

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