

ON DIFFERENCES OF SEMICONTINUOUS FUNCTIONS  
AND PERFECT CLASSES

PETR POŠTA

**Abstract.** Let  $K$  be a metric space and  $f : K \rightarrow \mathbb{R}$  be a bounded function. H. Rosenthal and others showed in a series of papers that  $f$  can be written as a difference of two bounded semicontinuous functions on  $K$  if and only if its transfinite oscillations are bounded on  $K$ . We provide a generalization of this characterization to an arbitrary Hausdorff topological space. As an application, we provide an alternative proof of the result obtained by J. Saint Raymond that the class of differences of semicontinuous functions is perfect.

**MSC 2010.** 26A15.

**Key words.** Differences of semicontinuous functions, perfect classes of functions

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Received January 7, 2017

Accepted July 10, 2017

*Charles University*  
*Faculty of Mathematics and Physics*  
*Department of Mathematics*  
*Sokolovská 83*  
*186 75 Praha 8, Czech Republic*  
*E-mail: pposta@karlin.mff.cuni.cz*